Consortium for Systemic Risk Analytics

The Decision to Lever

Robert Anderson Stephen Bianchi Lisa Goldberg

University of California, Berkeley

CENTER FOR
RISK MANAGEMENT
RESEARCH

December 12, 2013

Outline

Outline

Outline

Outline

Leverage and systemic risk

Much has been said about the leverage-liquidity spirals that dominate the news amidst market chaos

Leverage and systemic risk

Much has been said about the leverage-liquidity spirals that dominate the news amidst market chaos

Less newsworthy, but perhaps as important, is a persistent source of instability in a levered strategy that has (to the best of our knowledge) gone unnoticed

The elements of a levered strategy

Source portfolio S

the fully invested portfolio to be levered

The elements of a levered strategy

Source portfolio S

the fully invested portfolio to be levered

Borrowing rate r^b

not the risk-free rate in practice

The elements of a levered strategy

Source portfolio S

the fully invested portfolio to be levered

Borrowing rate r^b

not the risk-free rate in practice

Target T

that determines...

The elements of a levered strategy

Source portfolio S

the fully invested portfolio to be levered

Borrowing rate r^b

not the risk-free rate in practice

Target T

that determines...

Leverage λ

assumed to be at least 1 for convenience

Return to a levered portfolio

In a single period:

$$\begin{aligned} r^{\mathbf{L}} &= \lambda r^{\mathbf{S}} - (\lambda - 1) r^{b} \\ &= r^{\mathbf{S}} + (\lambda - 1)\left(r^{\mathbf{S}} - r^{b}\right) \end{aligned}$$

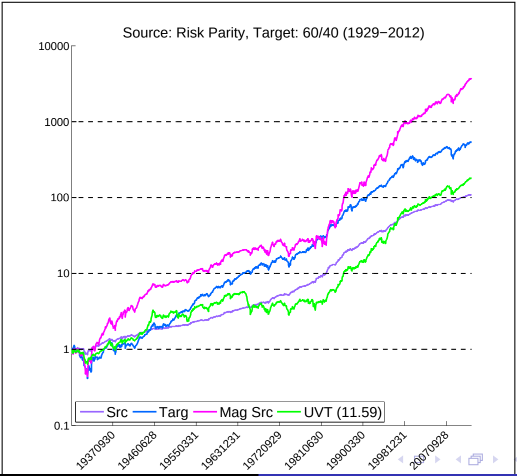
Return to a levered portfolio

Over many periods:

$$\begin{aligned} E\left[r^{\mathbf{L}}\right] &= E\left[r^{\mathbf{S}}\right] + E\left[(\lambda-1)(r^{\mathbf{S}}-r^{b})\right] + E\left[r^{\mathbf{TC}}\right] \\ &= \underbrace{E\left[r^{\mathbf{S}}\right] + E\left[\lambda-1\right]E\left[r^{\mathbf{S}}-r^{b}\right]} + \mathrm{cov}\left(\lambda, r^{\mathbf{S}}-r^{b}\right) + E\left[r^{\mathbf{TC}}\right] \\ &= \text{Magnified Source Return} + \text{Covariance} + \text{Trading Cost} \end{aligned}$$

CENTER FOR RISK MANAGEMENT RESEARCH

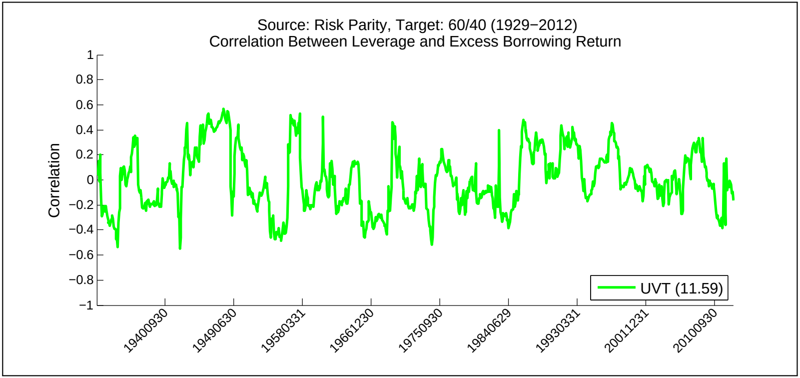
Cumulative returns to a levered risk parity strategy over an 84-year period



Decomposition of arithmetic expected return

	Contribution (annualized percent)
Magnified Source Return	9.72
Covariance	-1.84
Trading Cost	-1.03
$E\left[r^L\right]$	6.85

Three-hear rolling correlation between leverage and excess borrowing return



Decomposition of the covariance term

Covariance is a product of a correlation and two volatilities:

$$\mathrm{cov}\left(\lambda, r^{\mathbf{S}} - r^b\right) = \mathrm{corr}(\lambda, r^{\mathbf{S}} - r^b) \cdot \sigma(\lambda) \cdot \sigma(r^{\mathbf{S}} - r^b)$$

$$-1.84 = -.057 \cdot 7.7 \cdot 4.2$$

CENTER FOR RISK MANAGEMENT RESEARCH

Decomposition of the covariance term

Covariance is a product of a correlation and two volatilities:

$$\text{cov}\left(\lambda, r^{\mathbf{S}} - r^{b}\right) = \text{corr}(\lambda, r^{\mathbf{S}} - r^{b}) \cdot \sigma(\lambda) \cdot \sigma(r^{\mathbf{S}} - r^{b})$$
$$-1.84 = -.057 \cdot 7.7 \cdot 4.2$$

Empirically, in the examples we have looked at, the covariance term has been material driven over horizons of four to nine decades and it has been driven by a small magnitude correlation

CENTER FOR RISK MANAGEMENT RESEARCH

Decomposition of the covariance term

Covariance is a product of a correlation and two volatilities:

$$\text{cov}\left(\lambda, r^{\mathbf{S}} - r^b\right) = \text{corr}(\lambda, r^{\mathbf{S}} - r^b) \cdot \sigma(\lambda) \cdot \sigma(r^{\mathbf{S}} - r^b)$$
$$-1.84 = -.057 \cdot 7.7 \cdot 4.2$$

Empirically, in the examples we have looked at, the covariance term has been material driven over horizons of four to nine decades and it has been driven by a small magnitude correlation

Theoretically, over ultra long horizons, the covariance term tends to a limit that can be positive, zero or negative

CENTER FOR RISK MANAGEMENT RESEARCH

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...
  - ...and over horizons of several decades, it seems to be driven by a small magnitude correlation

CENTER FOR
RISK MANAGEMENT
RESEARCH

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...
  - ...and over horizons of several decades, it seems to be driven by a small magnitude correlation
  - Over much longer horizons, theory indicates it is consistently signed

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...
  - ...and over horizons of several decades, it seems to be driven by a small magnitude correlation
  - Over much longer horizons, theory indicates it is consistently signed
- Other important elements are:

CENTER FOR RISK MANAGEMENT RESEARCH

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...
  - ...and over horizons of several decades, it seems to be driven by a small magnitude correlation
  - Over much longer horizons, theory indicates it is consistently signed
- Other important elements are:
  - Magnified source return

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...
  - ...and over horizons of several decades, it seems to be driven by a small magnitude correlation
  - Over much longer horizons, theory indicates it is consistently signed
- Other important elements are:
  - Magnified source return
  - Trading cost

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...
  - ...and over horizons of several decades, it seems to be driven by a small magnitude correlation
  - Over much longer horizons, theory indicates it is consistently signed
- Other important elements are:
  - Magnified source return
  - Trading cost
  - Volatility drag

CENTER FOR RISK MANAGEMENT RESEARCH

Summary

- A key element of the return to a levered strategy is the covariance of leverage with excess borrowing return
  - Empirically, the covariance is highly unstable over horizons of three to five years...
  - ...and over horizons of several decades, it seems to be driven by a small magnitude correlation
  - Over much longer horizons, theory indicates it is consistently signed
- Other important elements are:
  - Magnified source return
  - Trading cost
  - Volatility drag
- Trading cost and volatility drag are present in all strategies, but they are exacerbated by leverage

Risk parity specification

- Two asset classes: US Equity and US Treasury Bonds
- Rebalance monthly to:
  - Equalize estimated volatility of the two asset classes
  - Lever to a volatility target of 11.59%
- Volatility is estimated with a 36-month rolling window
- Borrowing is at the three-month Eurodollar deposit rate
- Horizon: January 1929–December 2012

CENTER FOR RISK MANAGEMENT RESEARCH

Risk parity return decomposition

Table 1: Performance Attribution

Sample Period: 1929-2012 Source: Risk Parity, Target: 60/40 r^b = 3M-EDR, with trading costs		UVT (11.59%)
$\mathbb{E}[\mathbf{r^S}]$ (gross of trading costs)		**5.75**
$\mathbb{E}[\lambda - 1]$	2.66	
$\mathbb{E}[r^S - r^b]$	1.49	
$\mathbb{E}[\lambda - 1] \cdot \mathbb{E}[r^S - r^b]$		3.97
$\mathbb{E}[\mathbf{r^S}] + \mathbb{E}[\lambda - \mathbf{1}] \cdot \mathbb{E}[\mathbf{r^S} - \mathbf{r^b}]$		**9.72**
$\sigma(\lambda)$	7.7212	
$\sigma(r^S - r^b)$	4.2219	
$\rho(\lambda, r^S - r^b)$	-0.0566	
$\mathbf{C}ov(\lambda, r^S - r^b)$		-1.84
$-\mathbb{E}[r^{TCS}]$		-0.07
$-\mathbb{E}[r^{TCL}]$		-0.96
$\mathbb{E}[\mathbf{r^L}]$		**6.85**
$(1 + \mathbb{E}[r^L]/1200)^{12}$	1.0707	
$\exp(-\sigma^2_{r^L}/2)$	0.9934	
$[(1 + \mathbb{E}[r^L]/1200)^{12} \cdot \exp(-\sigma^2_{r^L}/2) - 1] \cdot 100 - \mathbb{E}[r^L]$		-0.48
Approximation Error		0.00
$\mathbb{G}[\mathbf{r^L}]$		**6.37**

CENTER FOR RISK MANAGEMENT RESEARCH

Trading cost

The drag on return from trading costs comes from turnover in the source portfolio and turnover due to leverage:

$$r^{\mathbf{TC}} = r^{\mathbf{TCS}} + r^{\mathbf{TCL}}$$

Determination of $r^{\mathbf{TC}}$ is a fixed point problem since trading costs are paid out of investor equity

Geometric return

- Geometric return, and not arithmetic return, determines long-term performance
- Higher volatility places a greater on cumulative return...
- ... and leverage magnifies volatility

$$G\left[r\right] \sim \left(1 + E\left[r\right]\right) e^{-\frac{\operatorname{var}(r)}{2}} - 1$$

CENTER FOR RISK MANAGEMENT RESEARCH

Looking ahead

- The choice of an 11.59% volatility target is not an accident
- That value is the volatility of the 60/40 strategy over the study period
- We relied on perfect foresight to set the target..
- ... and the result is highly sensitive to the choice