Asset-Liability Modeling in National Pension Plans

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Cambridge, MA
May 27, 2015

GXN-0715

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- Pension plans ought to manage

$$\text{Surplus} = \text{Assets minus Liabilities}$$

rather than Assets alone

Pension Plan Types

Prefunded
(Employer)

Partially
Prefunded
(National)

Pay As
You Go
(National)

- Ability to Invest Assets to Offset Liability Risk
- Importance of Age Distribution

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Partially Prefunded:

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- No consensus on how to do ALM

Proposed Stability Notion in Partially Prefunded Plans: Stable Funded Ratio

Theorem (Stable Funded Ratio)

Define the Stable Funded Ratio Contribution Rate as

$$\mathrm{SFRCR}_t = \mathrm{FR}_t \times \mathrm{PNC}_t + (1 - \mathrm{FR}_t) \times \mathrm{PAYGC}_t \tag{1}$$

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then the Funded Ratio is stable

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- Equation (2) is *exactly* the Sharpe-Tint Surplus [11] with κ set equal to the target Funded Ratio

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  - Allows planning for deteriorating demographics
  - Takes into account the effect of macroeconomic variables

References I

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