Network Connectivity, Systematic Risk and Diversification

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Introduction

- There is a general agreement on the traditional decomposition of an asset (portfolio) total risk into systematic and idiosyncratic components following the CAPM model
- Systematic risks comes from the dependence of returns on common factors
- Idiosyncratic risks are asset-specific
- But...
- There is also a recent consensus on the existence of systemic risks

Introduction

- Systemic risk definition: any set of circumstances that threatens the stability of, or public confidence in, the financial system
- Systemic risk is a function of a **system**
- Systemic risk arises endogenously from a system
- Systemic risk is a function of connections between and the structure of financial institutions or, more generally, between the companies and/or economic sectors

(Challenging) Research questions

- **What is the impact of network connectivity on the asset return loading to common systematic factors?**
- **Does network connectivity endanger the power of diversification?**

Measuring systemic links

- An increasing literature in economics investigates the role of interconnections between different firms and sectors, functioning as a potential propagation mechanism of idiosyncratic shocks throughout the economy.
- Canonical idea: Lucas (1977), among others, that states that such microeconomic shocks would average out and thus, would only have negligible aggregate effects. Similarly, these shocks would have little impact on asset prices.
- However:
- Acemoglou et al. (2011) use network structure to show the possibility that aggregate fluctuations may originate from microeconomic shocks to firms. Such a possibility is discarded in standard macroeconomics models due to a "diversification argument".
- Shock propagation in static networks Horvath (1998, 2000), Dupor (1999), Shea (2002), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2011).

Pricing and diversification in multifactor models

- General multi-factor model for a $k-$dimensional vector of risky assets (m risk factors)

$$\mathbf{R}_t = \mathbb{E}\left[\mathbf{R}_t\right] + B\mathbf{F}_t + \epsilon_t \qquad (1)$$

- Under equilibrium expected returns depend on the factor risk premiums Λ

$$\mathbb{E}\left[\mathbf{R}_t\right] = r_f + B\Lambda \qquad (2)$$

- Standard total risk decomposition

$$\begin{aligned} VAR\left[\mathbf{R}_t\right] &= BVAR\left[\mathbf{F}_t\right]B' + VAR\left[\epsilon_t\right] \qquad (3) \\ \Sigma_R &= B\Sigma_F B' + \Omega \qquad (4) \end{aligned}$$

Pricing and diversification in multifactor models

- Portfolio risk decomposition (ω being a $k-$dimensional vector of portfolio weights)

$$\begin{aligned} VAR\left[\omega'\mathbf{R}_t\right] &= \omega' BVAR\left[\mathbf{F}_t\right] B'\omega + \omega' VAR\left[\epsilon_t\right]\omega & (5) \\ \omega'\Sigma_R\omega &= \omega' B\Sigma_F B'\omega + \omega'\Omega\omega & (6) \end{aligned}$$

- Diversification benefit

$$lim_{k\to\infty}\omega'\Omega\omega = \nu \quad (7)$$

- Special case: uncorrelated idiosyncratic shocks with average variance $\bar{\sigma}^2$

$$lim_{k\to\infty}\omega'\Omega\omega = \frac{1}{k}\bar{\sigma}^2 = 0 \quad (8)$$

An augmented multifactor model with network connectivity

- Network connectivity represents contemporaneous relations across assets that co-exists with the dependence on common systematic risk factors
- Network connections are contemporaneous relations across endogenous variables

$$A\left(\mathbf{R}_t - \mathbb{E}\left[\mathbf{R}_t\right]\right) = B\mathbf{F}_t + \epsilon_t \tag{9}$$

- The simultaneous equation system above is not identified unless we impose some restriction; k number of assets is much larger than m the number of factors
- **Assumption 1:** the idiosyncratic shocks are uncorrelated, that is Ω is a diagonal matrix
- Assumption already taken into account in multi-factor models
- Key idea: Predetermined networks provide information on the existence of links across assets and on the strength/intensity of the link
- Networks can be represented as spatial matrices

A general framework with systemic and systematic risks

- By means of spatial matrices we can impose a structure on matrix A and rewrite the simultaneous equation system

$$\begin{aligned} A &= I - \rho W && (10) \\ (\mathbf{R}_t - \mathbb{E}[\mathbf{R}_t]) &= \rho W (\mathbf{R}_t - \mathbb{E}[\mathbf{R}_t]) + B\mathbf{F}_t + \epsilon_t && (11) \end{aligned}$$

- The coefficient ρ represents the impact coming from neighbours (by now we assume it is a scalar) and W represents neighbour relationships
- This simultaneous equation system corresponds to a Spatial Auto Regression Panel model where the covariates (risk factors) are common across all subjects (at least in a simplified representation)

Our contributions

- Take a multifactor model and focus on both pricing and diversification effects
- Measure systemic links starting from a network capturing causality relations
- Show that network links act as an inflating factor on the asset loadings to the common factors
- Show that network elements impact on both the systematic and idiosyncratic risk components

Network connectivity impact on expected returns

- We therefore have:

$$(I - \rho W)(\mathbf{R}_t - \mathbb{E}[\mathbf{R}_t]) = B\mathbf{F}_t + \epsilon_t \tag{12}$$

- It holds that

$$(I - \rho W)^{-1} = I + \rho W + \rho^2 W^2 + \rho^3 W^3 \dots \tag{13}$$

- Therefore the model corresponds to

$$\mathbf{R}_t = \mathbb{E}[\mathbf{R}_t] + B\mathbf{F}_t + \sum_{j=1}^{\infty} \rho^j W^j B\mathbf{F}_t + \eta_t \tag{14}$$

Network connectivity impact on expected returns

- Under equilibrium the expected returns equal

$$\mathbb{E}\left[\mathbf{R}_t\right] = r_f + B\Lambda + \sum_{j=1}^{\infty} \rho^j W^j B\Lambda \tag{15}$$

- where Λ represents systematic factor risk premia
- Therefore, as $\rho > 0$ and elements in W are positive, the presence of network/systemic links inflates the loading to the factors with an impact on the asset expected returns
- Expected returns increase as a consequence to an increase in ρ or a change in W with subsequent effects on prices

Network connectivity impact on risk

- We can play around this decomposition to recover a more insightful one

$$
\begin{aligned}
VAR\left[\mathbf{R}_t\right] &= A^{-1}B\Sigma_F B'\left(A^{-1}\right)' + A^{-1}\Omega\left(A^{-1}\right)' && (16)\\
&= \bar{B}\Sigma_F\bar{B}' + \mathcal{A}\Omega\mathcal{A}' && (17)\\
&= \bar{B}\Sigma_F\bar{B}' + \mathcal{A}\Omega\mathcal{A}' \pm B\Sigma_F B' \pm \Omega && (18)\\
&= \underbrace{B\Sigma_F B'}_{i} + \underbrace{\Omega}_{ii} + \underbrace{\left(\bar{B}\Sigma_F\bar{B}' - B\Sigma_F B'\right)}_{iii} + \underbrace{\left(\mathcal{A}\Omega\mathcal{A}' - \Omega\right)}_{iv} && (19)
\end{aligned}
$$

- We have thus four terms in the risk decomposition
  - i The structural systematic component
  - ii The structural idiosyncratic component
  - iii The network connectivity impact on the structural systematic component
  - iv The network connectivity impact on the idiosyncratic component

Systemic links impact on risk

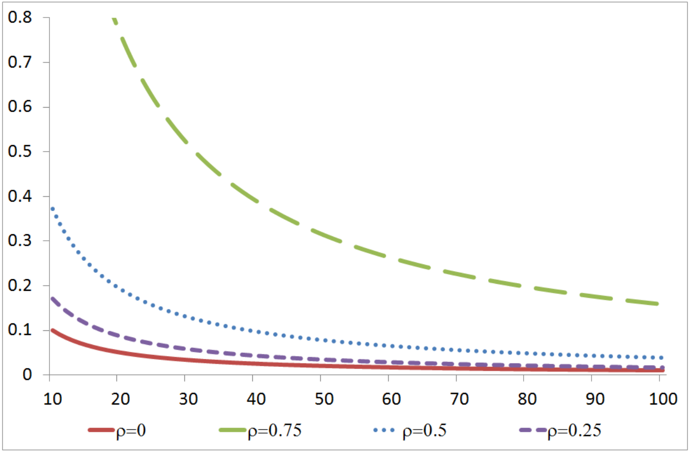
- This has effects on the diversification benefits which can be analytically derived in a special case
- Consider K uncorrelated idiosyncratic shocks with average variance $\bar{\sigma}^2$ and a W matrix where all assets are linked to each other, we have

$$lim_{K\rightarrow\infty}\omega'\mathcal{A}\Omega_\eta\mathcal{A}'\omega = \frac{K+\rho^2}{\left(K+\rho\right)^2\left(\rho-1\right)^2}\bar{\sigma}^2 = 0 \quad (20)$$

- Diversification benefits still present but the decrease of the idiosyncratic component of the portfolio variance is much slower

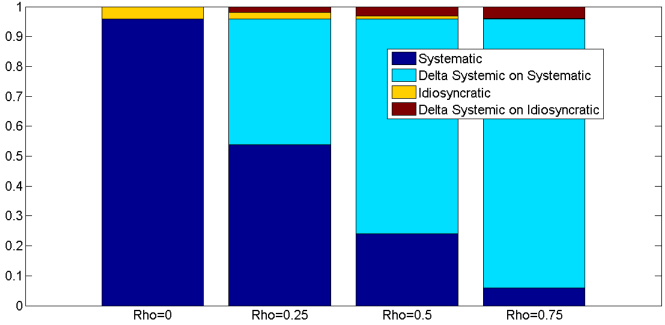
Systemic links impact on risk

- Portfolio idiosyncratic risk across different ρ levels and increasing number of assets. The case $\rho = 0$ corresponds to the absence of spatial links and is the standard result for diversification benefits.



Simulated examples

- $1/N$ portfolio variance decomposition across different values of ρ with the same "random" matrix W (relative decomposition)



Future developments

- Generalize the model in four directions
- First by adding heterogeneity to the asset reaction to the systemic links

$$A = I - \mathcal{R}W \tag{21}$$
$$\mathcal{R} = diag\left(\rho_1, \rho_2, \ldots, \rho_N\right) \tag{22}$$

- Second, by allowing for a time-variation in the W matrix

$$A \Rightarrow A_t = I - \mathcal{R}W_{g(t)} \tag{23}$$

- In this way the matrices $W_{g(t)}$ capture the dynamic in the links across assets (the evolution of the newtork), while the spatial coefficients included in $\mathcal{R}$ represent the impact on each asset of the neighbours, and are assumed to be time-independent
- The $W_{g(t)}$ matrices are also assumed to be known at time t and thus we condition the model estimation and the analysis to their availability

Future developments

- We assume the evolution of $W_{g(t)}$ is smooth or acts at a time scale much lower than that affecting the evolution of asset returns (i.e. for monthly returns the W change with, say, a yearly frequency); to highlight this aspect we index the spatial links matrices to a function of time
- Deeper evaluation of risk factor exposure and pricing implications; analysis of diversification impact of network exposure
- Third: time-variation in the ρ coefficients
- Fourth: different design of the W matrices taking into account the strength of the relation across subjects