Contract as Automaton
The Computational Representation of Financial Agreements

Mark D. Flood (OFR)
and
Oliver R. Goodenough (Vermont Law School and OFR)

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Additional debt to others in Computational Law

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Measuring Financial Contracts

- Instruments
- Legal Entities
- Relationships
- Positions & Portfolios
- Messages
- Transactions
Computational Contracts

• Key insights
  – Financial contracts are central to understanding the financial system
  – Financial agreements aid coordination by alleviating “social uncertainty”
  – Formal modeling of structure makes it accessible to programmatic analysis
  – Financial contracts are structured internally as state-transition systems

• Proof of concept
  – A single, simple loan agreement, stated in traditional legalese
  – Underlying structure of that agreement as a discrete finite automaton (DFA)
  – Three (interchangeable) representations of the structure:
    • Graphical
    • Tabular
    • Regular expression
  – Discretization and finiteness are crucial to managing complexity
A “Toy” Loan Agreement

Simple two-page loan contract

1. The Loan: $1000, June 1, 2014
2. Repayment:
   • Payment 1, due June 1, 2015: $550
   • Payment 2, due June 1, 2016: $525
3. Representations and Warranties
4. Covenants
5. Events of Default:
   • Borrower fails to make timely payment
   • Reps or warranties prove untrue
   • Borrower fails any covenants
   • Borrower files for bankruptcy
6. Acceleration on Default
7. Choice of Law
8. Amendments and Waivers
9. Courts and Litigation
10. Time of the Essence; No Pre-Payment
11. Notices
• A deterministic finite automaton (DFA) is defined by a 5-tuple:
  – Finite set of states \((Q)\)
  – Finite set of input symbols (information/events) called the alphabet \((\Sigma)\)
  – Transition function \((\delta : Q \times \Sigma \rightarrow Q)\)
  – Start state \((q_0 \in Q)\)
  – Set of accept (end) states \((F \subseteq Q)\)

• Three representations (at least):
  – Graphical (depiction of states and transitions among them)
  – Lists (of \(Q\), \(\Sigma\) and \(\delta\))
  – Regular expression (shorthand grammar of acceptable event sequences)
DFA as a chain of event and consequence:

- Start state \( q_0 \) at the top
- Terminal states (3) at bottom
- “Happy” or intended path traced in green
- More “interesting” ramifications traced in black
From the state [Pmt 1 accruing], four transitions are possible:

- Three types of default:
  - Reps/warranties
  - Covenant
  - Bankruptcy
- Due date for first payment arrives <June 1, 2015 passes>
**Representation II: Tabular**

### State Space (27)

- **Q**

### Event Alphabet (20)

- **Σ**

### Transitions (45)

- **δ**

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**Table:**

<table>
<thead>
<tr>
<th>ID</th>
<th>Label</th>
<th>Natural language event specification</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Contract signed</td>
<td>contract is signed in 3rd party title</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1 day passed since last event</td>
<td>June 1, 2014 passed</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>Money requested</td>
<td>Borrower given request for loan of $1,000</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>Loan set</td>
<td>A legal action is brought to enforce, interpret or otherwise deal with the agreement in the state courts of the state of New York located in New York county.</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>Notice of limitations</td>
<td>Notice of limitations on debt obligations in New York state is a court that has jurisdiction over the matter due to the amount of $1,000.</td>
<td>4</td>
</tr>
<tr>
<td>F</td>
<td>Principal advanced</td>
<td>Lender advances $1,000 no later than June 7, 2016.</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>June 1, 2016 advanced</td>
<td>Payment 1 due by June 1, 2016</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>Repayment</td>
<td>The borrower's rights to participate in a bankruptcy proceeding under applicable federal or state law</td>
<td>5</td>
</tr>
<tr>
<td>I</td>
<td>Covenant</td>
<td>The borrower's rights to participate in a bankruptcy proceeding under applicable federal or state law</td>
<td>5</td>
</tr>
<tr>
<td>J</td>
<td>Bankruptcy</td>
<td>The borrower's rights to participate in a bankruptcy proceeding under applicable federal or state law</td>
<td>6</td>
</tr>
<tr>
<td>K</td>
<td>Notice given</td>
<td>Notice given to borrower of a failure to make timely payment of an amount due to lender under the agreement</td>
<td>5</td>
</tr>
<tr>
<td>L</td>
<td>Notice given of general default</td>
<td>Notice given to borrower of a failure to make timely payment of an amount due to lender under the agreement</td>
<td>5</td>
</tr>
<tr>
<td>M</td>
<td>Payment default cured</td>
<td>A payment-related event of default is cured</td>
<td>5</td>
</tr>
<tr>
<td>N</td>
<td>General default cured</td>
<td>A non-payment-related event of default is cured</td>
<td>5</td>
</tr>
<tr>
<td>O</td>
<td>2 Days pass since last event</td>
<td>Two days have passed since last event occurrence</td>
<td>5</td>
</tr>
<tr>
<td>P</td>
<td>June 1, 2016 passed</td>
<td>Payment 2 is due on June 1, 2016</td>
<td>5</td>
</tr>
<tr>
<td>Q</td>
<td>Payment made late</td>
<td>Payment made late</td>
<td>5</td>
</tr>
<tr>
<td>R</td>
<td>Payment made late</td>
<td>Payment made late</td>
<td>5</td>
</tr>
<tr>
<td>S</td>
<td>Payment made late</td>
<td>Payment made late</td>
<td>5</td>
</tr>
<tr>
<td>T</td>
<td>Contract or modify</td>
<td>Contract in the form is canceled because of modification or termination by mutual agreement of the parties</td>
<td>5</td>
</tr>
</tbody>
</table>

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**Image Source:** OFR analysis

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Representation III: Regular Expression

\[ A(B | CB[ED]) | \]

\[ ACF(G(BK) ?)QPR | \]

\[ ACF([HIJ]LN) *(GBK| [HIJ]L)O(S | B[DES]) | \]

\[ ACF(G(BK) ?)Q([HIJ]LN) *(PBK| [HIJ]L)O(R | B[RED]) \]

**Rapid demise**

**Happy path**

**Unhappy 1**

**Unhappy 2**
## Full Transition Matrix

|   | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
| start | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| q0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| q1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| q2 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| q3 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| P1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| P2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| P3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| P4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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Implications – Complexity

• **Basic Results on Complexity**
  – DFAs enforce the Markov (or Myhill-Nerode) property – *state is “memoryless”*
    • The DFA “lives in the moment” – all transitions are one-step-ahead actions
  – Computational complexity is manageable:
    • Constrained by the Myhill-Nerode condition
    • Measurable by the descriptional complexity of the regular expression
  – The law appears to have evolved this constraint organically
    • Sorcerer’s Apprentice problem

• **Assessing Complexity**
  – The complexity of actual contracts is (in theory) rigorously measurable
  – The computational “inefficiency” of a contract is measurable:
    • Measure the contract’s actual complexity, $C$
    • Reduce the contract’s DFA to its theoretical minimum and measure that complexity, $C^*$
    • The difference, $\Delta C = (C-C^*)$, is a measure of “unnecessary” complexity
Next Steps - Modeling

• **Nondeterministic Finite Automata (NFAs)**
  – Standard extension of DFAs
  – Identical expressiveness, but:
    • Additional flexibility in representation
    • Therefore typically more compact

• **Transducers**
  – Extend the DFA representation to emit events
    • For example, cross-default clauses
  – Contracts that listen to other contracts
  – Systemic implications
  – Two standard cases:
    • Moore machine – Transition output event *cannot* depend on triggering input event
    • Mealy machine – Transition output event *can* depend on triggering input event
Next Steps - Empirical

- **Real Contracts**
  - International Swap Dealers Association (ISDA) – OTC swaps
  - International Foreign Exchange Master Agreement (IFEMA) – Spot FX
  - Standardizing contingency clauses
    - Work is underway on ISDA master agreements

- **Event measurement**
  - Principles for defining measurable events to support DFA representations
  - Tools for feature extraction
  - Contractual completeness (relative to the event space)
  - Contractual coherence (relative to the event space)
Thanks!