Contract as Automaton

The Computational Representation of Financial Agreements

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Abstract:
We show that the fundamental legal structure of a well written financial contract follows a state-transition logic that can be formalized mathematically as a finite-state machine (a.k.a. finite-state automaton). The automaton defines the *states* that a financial relationship can be in, such as "default," "delinquency," "performing," etc., and it defines an *alphabet* of events that can trigger state transitions, such as "payment arrives," "due date passes," etc. The core of a contract thus describes the rules according to which different sequences of event arrivals trigger particular sequences of state transitions in the relationship between the counterparties. By conceptualizing and representing the legal structure of a contract in this way, we expose it to a range of powerful tools and results from the theory of computation. These allow, for example, automated reasoning to determine whether a contract is internally coherent, and whether it is complete relative to a particular event alphabet. We illustrate the process by representing a simple loan agreement as an automaton.

I. Introduction

Computing has revolutionized one sector of human activity after another over the past half century. Until recently, however, this process had limited impact on law. The power of automation *has* made human lawyers more efficient in their traditional roles of drafting, research, negotiation and advocacy, but the human brain has remained the central processor in understanding and applying the logical structure that informs legal rules and obligations. We argue that legal logic at a deeper level aspires to the requisite characteristics of rigorous definition and internal consistency that should make it accessible to formal computational tools. Lessig's famous formulation is that "code is law".[1] We invert this formulation and assert that, in fact, law is computation. In particular, we argue that a well-written financial contract follows a state-transition logic (usually implicit). Moreover, this logic is typically sufficiently streamlined and regular that we can represent it with a simple but important computational formalism, namely a "finite state machine." We demonstrate this process with a concrete example in Sections IV and V below.

We argue that a well written financial contract is a "finite state machine" in a formal sense, and demonstrate the process in Sections IV and V below.[2] That is, the structure of a contract employs legalese – a relatively precise form of natural language expression – to encode a finite set of ***states*** that can describe the relationship between the counterparties at a given point in the life of the contract. Moreover, the contract also encodes explicit ***transition*** rules for shifting the relationship from one state to another, based on the realization of certain predefined ***events***, such as performance by the counterparties themselves or the occurrence of particular contingencies that may be within or without their control. Locating legal rules in computational form will have early benefits in the fields of private contracting in general and of financial contracting in particular.

The theory of automata is fundamental to computer science. By formalizing contracts according to rules of the literature on computability, we expose them to a wealth of powerful machinery from that domain, such as programmatic testing for (legal) completeness and (computational) complexity, and tools for simplification, visualization, and even the automated generation of legalese. Conversely, forcing a contract to adhere to the prerequisites of the state machine model – especially the finiteness of states and events, and the independence of states from one another – imposes valuable discipline on the legal artisan, and suggests a normative argument for *how* contracts should be crafted.

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[1] Lessig, Lawrence. *Code: And Other Laws of Cyberspace*. New York, N.Y.: Basic Books, 2000.

[2] There are many excellent textbook treatments of the theory of computation and finite state machines, including Sipser, Michael. *Introduction to the Theory of Computation*. 2nd ed. Boston, MA: Thompson Course Technology, 2006.and Kozen, Dexter C. *Automata and Computability*. New York, N.Y.: Springer, 1997..; and Rosenberg, Arnold L. *The Pillars of Computation Theory State, Encoding, Nondeterminism*. New York: Springer, 2010. For an overview, see the lectures by Shai Simonson, at " Theory of Computation." ADUniorg RSS. May 14, 2013. Accessed December 12, 2014. http://www.aduni.org/courses/theory/.

This paper has several goals. The first is relatively simple: to report the results of an implementation project designed to test this thesis about contracts. We have restated the logical structure of a simple financial contract completely in the formal computational terms of a "deterministic finite automaton" ("DFA"), a particular kind of state machine. The second, more expansive, goal is to suggest how financial contracting more generally could be conceived through a computational lens. A third goal, more ambitious still, is to explore some of the implications that grow from viewing contracts and other areas of the law as a system of computation. Before describing our project, however, we will provide some background on financial contracts and on computational approaches to law.

II. Economic Importance and Structure of Contracts

Contracts are an important tool for coordinating economic activity. As far back as the 18th century Adam Smith noted the implications for human wellbeing of the opportunities for productive sociality that underlie the gains of trade and specialization.[3] A principal lesson of game theory, however, is that the pathways to productive social interaction are often blocked by strategic dilemmas.[4] The opportunities for defection, predation, and other short-term strategies of selfish exploitation can be daunting obstacles to achieving the potential rewards of cooperation.[5] Much of game theory concerns social behavior in the absence of enforceable commitments, so that players must typically fall back on the subset of behaviors that are incentive-compatible. The famous prisoners' dilemma is the classic example, with its self-fulfilling Nash equilibrium of missed opportunities and productivity foregone.

We need not be prisoners of the prisoners' dilemma, however. Humans have the capacity to design alternative game structures that avoid many of the traps of defection. These structures can be variously instantiated, for example, informally, through reputation, psychology, or cultural practice; or formally, through physical structures or through processes like law. Such arrangements can be termed "institutions" – and institutions matter greatly.[6]

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[3] Smith, Adam, and Edwin Cannan. *An Inquiry into the Nature and Causes of the Wealth of Nations*. Canaan ed. New York: Modern Library, 1937. In particular, consider his example of the division of labor in a pin factory (Book I, Chapter 1), and his discussion of the human propensity to "truck, barter, and exchange on thing for another" (Book I, Chapter 2) that makes a division of labor possible.

[4] The seminal reference on game theory is Neumann, John, and Oskar Morgenstern. *Theory of Games and Economic Behavior,*. Princeton: Princeton University Press, 1944. For more recent treatments, see Fudenberg, Drew, and Jean Tirole. *Game Theory*. Cambridge, Mass.: MIT Press, 1991, or Gintis, Herbert. *The Bounds of Reason: Game Theory and the Unification of the Behavioral Sciences*. Princeton, N.J.: Princeton University Press, 2009.

[5] Goodenough, Oliver R., "Values, Mechanism Design, and Fairness", in Zak, Paul J. *Moral Markets the Critical Role of Values in the Economy*. Princeton: Princeton University Press, 2008. 228-255.

[6] On reputation and other informal relationship structures, see MacLeod, W Bentley. "Reputations, Relationships, and Contract Enforcement." *Journal of Economic Literature*, 45(3): 595-628 (2007) **,** and Mailath, George Joseph, and Larry Samuelson. *Repeated Games and Reputations: Long-run Relationships*. Oxford: Oxford University Press,

A principal function of law is providing institutional frameworks for productive sociality that are built around promises of enforcement. A comparison of game theory with the economic theory of contracts is instructive in this context. Contracts augment the relatively simple setting of game theory by providing an external enforcement mechanism.[7] Enforcement provides additional tools for participants to commit to their own future actions, vastly expanding the potential scope of coordination. With the threat of punishment in the background, more credible promises are possible. As a result, the central questions for contract theory can extend to the practical considerations that affect the quality of promises, such as the incentives to hide information or shirk duties and the capacity of participants to verify their counterparties' actions, or reliably certify their own.

Some non-contractual enforcement frameworks, such as criminal laws codified in statutes or promulgated regulations, are "pre-set", in the sense that they are made generally applicable across a wide swath of actors in society and have limited space for customization to a particular set of facts. The law of contracts, by contrast, is specifically intended to create open-ended institutional design spaces, where a few private parties – at least two – can come together and create binding obligations of their own. As such, it is widely recognized that contracts have particular value as vehicles for seeking arrangements that will be Pareto enhancing, i.e. leading to gains for at least one of the parties, while the others are at least no worse off from the interaction.[8]

2006.. On the evolution of more formal structures and institutions, see Goodenough, Note 5 Supra. Also see Crawford, Sue E. S., and Elinor Ostrom. "A Grammar of Institutions." *The American Political Science Review*, 1995, 582-600. Ostrom, Elinor. "Enhancing the Evolution of Institutions for Collective Action." *Social Evolution Forum* (March 26 2012), available at http://socialevolutionforum.com/2012/03/26/elinor-ostrom-enhancing-the-evolution-of-institutions-for-collective-action/; Ostrom, Elinor, and Roy Gardner. *Rules, Games, and Common-pool Resources*. Ann Arbor: University of Michigan Press, 1994.

[7] Game theory typically assumes that players follow the rules of the game; else the space of possible strategies is impossibly large. There is a strand of the game theory literature that uses finite automata as a formalism for specifying players' strategies in repeated games; see, for example Abreu, Dilip, and Ariel Rubinstein. "The Structure of Nash Equilibrium in Repeated Games with Finite Automata." *Econometrica*, 1988, 1259-281., and Binmore, Kenneth G., and Larry Samuelson. "Evolutionary Stability in Repeated Games Played by Finite Automata." *Journal of Economic Theory*, 1992, 278-305. In these models, the automata are heuristic rules for the individual players' strategies, and not descriptions of the *relationship* between the agents. For an excellent overview of the economic theory of contracts, see Bolton, Patrick, and M. Dewatripont. *Contract Theory*. Cambridge, Mass.: MIT Press, 2005.

[8] Trebilcock, M. J. *The Limits of Freedom of Contract*. Cambridge, Mass.: Harvard University Press, 1997. There are challenges in this story, however, that can make the intervention of legal constraints on consent appropriate in many circumstances. See, e.g., Drobac, Jennifer A., and Oliver R. Goodenough. "Exposing the Myth of Consent" *Indiana Health Law Review*, 2015 forthcoming; Goodenough, Oliver R. "Governance for Cloud Computing: The Role of Public and Private Rulemaking in Promoting the Growth of a New Industry" (October 19, 2013). Vermont Law School Research Paper No. 34-13. Available at SSRN: http://ssrn.com/abstract=2342594 or http://dx.doi.org/10.2139/ssrn.2342594.

An important challenge of this approach is contractual completeness, or the range of contingencies that a contract is able to specify before the fact, such that events are verifiable after the fact. As we show in Sections IV and V below, this event space plays a crucial role in the design and specification of a contract. Contracts that support coordinated behavior across a wider range of eventualities are generally more useful.

Financial Contracts

The financial sector is particularly dependent on contracts as the basis of productive exchange.[9] Contracts are central to the economic life of financial markets; they define detailed relationships between counterparties, itemizing what each promises to do under a pre-specified list of important contingencies. In this way, contracts provide an important coordination mechanism that enables many transactions and other economic activities that would otherwise be impracticable. By enmeshing market participants in an intricate web of obligations, often with unforeseen interactions across contracts, financial agreements are also a fundamental contributor to the complexity and possible fragility of the system.

Consider the explosion in asset securitization that preceded the recent boom and bust in mortgage markets. This surge was enabled in part by advances – often apocryphal – in credit risk modeling that allowed underwriters to sell new mortgages assets (typically with third-party credit protection) into a securitization pipeline, thus clearing the balance sheet for additional lending.[10] The fact that many of these originations sidestepped the traditional underwriting diligence, thus populating the securitization trusts with flawed loans, is a cautionary tale. We speculate (perhaps optimistically) that adoption of the techniques of contract automation that we foresee in this paper might have ameliorated some of these excesses.

Contract formation is a matter of negotiation and specification of the "terms" of the agreement. Most exchange-based transactions use standardized contracts defined and settled by the exchange's clearinghouse. Even over-the-counter markets rely heavily on relatively standardized forms such as the 1992 and the 2002 Master Agreements promulgated by the International Swaps and Derivatives Association, Inc. (ISDA) for derivatives and swaps, and the 1997 International Foreign Exchange Master Agreement (IFEMA) promulgated by a consortium of foreign exchange specialty organizations.[11]

[9] For example, securities markets and securitizations are two areas of financial activity dominated by arms-length contractual relationships. These two areas have experienced supranormal growth over the past half-century. See Greenwood, Robin, and David Scharfstein. 2013. "The Growth of Finance." *Journal of Economic Perspectives*, 27(2): 3-28.

[10] See, generally, Taub, Jennifer. *Other People's Houses: How Decades of Bailouts, Captive Regulators, and Toxic Bankers Made Home Mortgages a Thrilling Business*. New Haven, CT: Yale University Press, 2014.

[11] The ISDA agreement is available at "ISDA Agreement." ISDA Bookstore. January 1, 2013. Accessed December 12, 2014. http://www.isda.org/publications/isdamasteragrmnt.aspx. The foreign exchange agreement is available at

Standardization is approximate under these master agreements, because each provides explicit opportunities within the standard framework for customization to meet the context of the particular transaction.

The more general case of customized "structured" investment products or "bespoke" agreements presents greater challenges both for issuers, who must craft novel contractual clauses, and for investors, who must interpret their ramifications. Bankers Trust's "LIBOR squared" contract of the 1990s illustrates some of the hazards of such customizations. The deal presented clients with what David Rowe has called "gratuitous complexity," a payoff that was both asymmetric and nonlinear function of the London interbank offered rate (LIBOR).[12] More recently, structured collateralized debt obligations (CDOs) backed by subprime mortgage loans offer another cautionary tale. A back-of-the-envelope calculation reveals the practical impossibility of traditional (i.e., human) diligence in analyzing these deals: a "simple" CDO involving bonds from 100 mortgage-backed security pools, each of which holds 100 mortgages, each with 100 pages of documentation, would require the analysis of one million pages of documentation. Moreover, many subprime CDOs were restructured into CDO^2s, CDO^3s, etc., further compounding the challenge. Indeed, a post-mortem empirical analysis by Cordell, Huang, and Williams (2011) reveals serious mispricing problems for structured subprime CDOs in the runup to the crisis, suggesting that appropriate diligence never occurred.[13]

Financial contracts, broadly defined, involve exchanges in money, securities, and securitized obligations for other goods and services, such as futures in petroleum or agricultural products. While there is considerable variation and specialization, such contracts also frequently share common structural elements. Agreements for equity style rights of "ownership" carry additional complications in that the often specify measures for participating in the governance of an organization.[14] We do not consider

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"The 1997 International Foreign Exchange Master Agreement (IFEMA)." Federal Reserve Bank of New York. January 1, 1997. Accessed December 12, 2014. http://www.newyorkfed.org/fmlg/ifema.pdf.

[12] See Rowe, David M. "Risk Management Beyond VaR," *EDPACS: The EDP Audit, Control, and Security Newsletter*, 2013, 48(1), 1-28. One client, Gibson Greetings, sued Bankers Trust after its plain-vanilla swap agreements were replaced with a LIBOR-squared variant. For a discussion of the Gibson Greetings case, see Overdahl, James, and Barry Schachter. "Derivatives Regulation and Financial Management: Lessons from Gibson Greetings." *Financial Management*: 68-78. For legal background on the case, see Lynn, David M. "Enforceability of Over-the-Counter Financial Derivatives." *The Business Lawyer ABA*, 1994, 291-337. More generally, see Macey, Jonathan R. *The Death of Corporate Reputation: How Overregulation Has Destroyed Integrity on Wall Street*. Upper Saddle River, N.J.: Financial Times Press/Pearson Education, 2013.

[13] See Cordell, Larry and Huang, Yilin and Williams, Meredith, Collateral Damage: Sizing and Assessing the Subprime CDO Crisis (August 1, 2011). FRB of Philadelphia Working Paper No. 11-30. More generally, see Taub (2014), supra note 10.

[14] The characterization of equity instruments as "ownership" is widely practiced, but it is a generalization that can lead to conclusions about the rights of participants in such instruments that are at odds with those actually allocated by law to the holders of shares of stock. See, e.g. Stout, Lynn A. *The Shareholder Value Myth How Putting Shareholders First Harms Investors, Corporations, and the Public*. San Francisco: Berrett-Koehler, 2012.

their challenges directly in this paper. Table 1 lists some common elements of a typical financial agreement (*not* including equity-style rights of participating ownership).

Table 1: Ingredients of a typical non-equity-style contract

It is convenient (although not necessary) to think of a financial contract as combining a statement of the desired sequence of events and actions, with statements of the various commitments and resorts to be pursued if circumstances should go awry. We refer to the desired execution sequence under a contractual relationship as the "happy path."[15] Very roughly, the sections of Table 1 denoted "Counterparties" and "Basic obligations" compose the happy path; the sections denoted "Default provisions" and "Enforcement" cover the various unhappy paths that the relationship might follow.

Taken together, the elements in Table 1 can create significant complexity, by linking a number of variables and factors together in chains of event and consequence and by delineating varied outcomes depending on the factors involved. For instance, an agreement might make a distinction between, and treat differently, (i) a default under an obligation to make a payment and (ii) a default under a covenant (such as a promise to provide a particular financial report), each playing out differently across the logic of the contract. Complexity almost always increases when a transaction leaves the happy path of expected and timely fulfillment and enters the world of default, penalty and enforcement, a world we will discuss at greater length in Section V below.

Because a contractual relationship typically envisions a single happy path and many unhappy paths, much of the effort in drafting agreements must address the latter. In contracting, as in software development, planning for exceptional situations and component failures are crucial for a robust process. A powerful characteristic of the deterministic finite automata that we describe below is that

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In the context of governance, note the strides taken in "virtual firm formation." This development is gaining some traction, in theory, in the law, and in practice. See, e.g., Goodenough, Oliver R. "Digital Firm Formation", in Kaufman Task Force on Law, Innovation and Growth, *Rules for Growth, Promoting Innovation and Growth Through Legal* Reform. Kansas City, Missouri: Kauffman, 2011 and "Digital Operating Agreement". Silverflume, Nevada Secretary of State . Accessed December 14, 2014. https://nvsilverflume.gov/digitaloa/home. While equity agreements can be automated, the agreements in such cases have the added task of specifying processes for making decisions in the future in the face of uncertain circumstances beyond the capacity of the parties to currently anticipate. As such, they incorporate governance mechanisms, and contain challenges for computational representation beyond those of the more fully determined contracts of debt and exchange examined here.

[15] The notion of the "happy path" comes from the world of software testing, where it refers to the primary software execution path that provides the core, required functionality, and does not provide for error handling or other exceptional events. In other words, the happy path "comprises the sequence of activities executed if everything goes as expected." "Happy Path." XUnit Patterns. February 9, 2009. Accessed December 12, 2014. http://xunitpatterns.com/happy path.html. See, for example, "'Happy Path' Testing," Acquisition Archetypes. Software Engineering Institute, Carnegie Mellon University. January 1, 2009. Accessed December 13, 2014. http://www.sei.cmu.edu/library/assets/happy.pdf.

DFAs rigorously enforce a simplicity requirement that the history of a process, such as the evolution of a contractual relationship, must be fully captured by the specification of its current state. We argue that well drafted financial contracts should and do adhere to this rule. This discipline helps keep the overall complexity of the system within manageable bounds.

Traditional Contract Drafting and Its Limits

The task of the drafter of a traditional contract is to specify, generally through expressions of natural language, four core features: (i) actions to be taken (or refrained from) by a party (ii) salient events and occurrences in the world (both relating to the parties to the transaction and relating more generally); (iii) imbedded calculations for deriving quantitative values relevant to (i) from the information provided in (ii), and (iv) the logic strings that link (i), (ii), and (iii) together in chains of event and consequence.[16] Specialists in such natural language specifications (i.e. transactional lawyers) have evolved a set of linguistic tools and conventions– a version of legalese – reasonably well adapted to this task.

In financial contracting, tools and conventions have been tailored through repeat usage to encompass the elements of a financial contract described above. These tools, and the standardized forms in which they are often embedded, are sufficiently effective that trillions of dollars in value are transferred under such word-based agreements every business day.[17] Even at their best, however, transactions specified through such legalese have limitations. Natural language is prone to ambiguity and incompleteness, which causes potential uncertainty, particularly in extreme or unanticipated conditions. Sometimes this ambiguity is seen as a positive feature, particularly by the lawyer whose drafting created it, or the trader who can leverage it as a source of asymmetric information.[18] More often, particularly in the relatively well specified world of financial instruments, this is seen as a detrimental bug, with potentially negative consequences for all concerned.[19] This ambiguity can affect each of the four elements set out above. It

[16] This set of features is not just a property of contractual specification. At a level of some generality, a similar set of inputs, processing for calculation and logic, and the resulting outputs can describe the application of law in litigation and regulatory environments.

[17] The foreign exchange market alone accounts for over USD 5T in daily volume; see "Triennial Central Bank Survey Foreign Exchange Turnover in April 2013: Preliminary Global Results." Bank for International Settlements. April 1, 2013. Accessed December 13, 2014. http://www.bis.org/publ/rpfx13fx.pdf. As a point of reference, U.S. equities markets generate over USD 200B in daily trading volume; see Clark, Colin. "Whither Equity Volumes?" NYSE Exchanges. January 31, 2011. Accessed December 13, 2014. Greenwood and Scharfstein (supra, note 11), emphasize the long-term trend, which shows steadily increasing volumes.

[18] McCarty, L. Thorne. "AI and Law: How to Get There from Here." Rutgers School of Arts and Sciences - Computer Science. March 1, 1990. Accessed December 13, 2014. http://www.cs.rutgers.edu/~mccarty/research/rj90.pdf.

[19] It is commonplace for users to think of undesired software behavior as a "bug" and desired behavior as a "feature." Because users often differ in their preferences, this categorization can be ambiguous, and the phrase "it's not a bug, it's a feature, " is a standing joke among software developers. See Herzig, Kim, Sascha Just, and Andreas Zeller. "It's Not a Bug, It's a Feature: How Misclassification Impacts Bug Prediction." Software Engineering

is typically least helpful in the context of specifying any mandated numerical calculations and of specifying the logic of the chain of event and consequence.

Measurement of the "events" to be recognized is another area where good financial contracting can help resolve ambiguity. The definition of key events provides a crucial simplification that allows a contractual relationship to function. Arbitrators and adjudicators – and the parties themselves – cannot countenance the full nuance of every possible eventuality that might occur over the life of a contract. Instead, the infinite complexities of social reality are mapped – or "discretized" – into a finite set of possibilities, each of which should ideally be objectively measurable. For example, in the "toy" contract presented below, the borrower is allowed two days to cure an event of default. There is no further distinction regarding timing within that window; any cure occurring within those tolerances is treated as equivalent. In the context of DFAs, this finiteness of the event space is intimately tied to the finiteness of the state space, which is a requirement for some of the most powerful results in computation theory. It is noteworthy that the law evolved this same intrinsic elegance independent from (and prior to) computer science.

In addition to these ambiguity concerns, natural language specification is not open to fast, accurate parsing. When interpretation is necessary, legalese, even when not ambiguous, makes slow, tortuous reading, with the need to check and recheck the definitions, cross-references, exceptions, etc. in which the complexity is embedded. Lawyer brains, the computational mechanism of traditional contract interpretation, are expensive and subject to cognitive limitations. Even well-trained lawyers process such convoluted natural language strings, and the factual background under which they are operating, relatively slowly.

III. Computable Contracting and Computational Law

These and other limitations inherent in natural language as a tool for specifying the agreements in financial contracting have led us to conclude that progress in the functioning and safety of financial markets will be served by tools that allow more exact specification than those of natural language for the construction and description of financial obligations. Contracts specified according to the more exacting formalism of DFAs are amenable to programmatic analysis with standard tools for verifying model correctness and completeness, and assessing overall complexity.[20] For many core tasks in the interpretation and application of contractual terms, the binding constraint on legal productivity will no longer be the availability of competent lawyers to perform such parsing. These tasks can be automated, and the challenges will move to new frontiers.

Chair, Saarland University - Computer Science. March 23, 2014. Accessed December 12, 2014. https://www.st.cs.uni-saarland.de/softevo//bugclassify/.

[20] On formal techniques for model checking and finite-state verification, see Baier, Christel, and Joost Katoen. *Principles of Model Checking*. Cambridge, Mass.: MIT Press, 2008.

The advantages of rapid computation of event and consequence are particularly important when contemplating the outcomes mandated once a transaction goes off the "happy path". This is not just a problem for the particular contract, but can become systemic, for example, through domino cascades of interlinked default across firms and markets, effected through cross-default clauses. Whether at the level of individual transactions, of the portfolio characteristics of a particular firm or account, or of the financial system as a whole, the ability to represent the *entire originals* of the financial contracts at issue in completely computable form will greatly increase the ability to test for both firm-specific and systemic weakness in advance and to cope with the consequences of a firm-wide or systemic market event when it occurs.

A number of researchers and groups are working on the methods of contract automation and computability. Some are looking for means to allow machine intelligence to parse existing natural language contractual formulations. Because this approach fails to model directly the legal logic of the contract, it has significant limitations. If one were asked to mechanize travel in the 19th century, creating a machine that emulated the movements of a horse would not achieve all of the power possible in the new medium – the wheeled automobile was a better approach. [21] Making the flawed processes of natural language contracting the continued foundation for contract computability will be similarly limiting. Rather, we believe that many of the tasks and specifications delegated to natural language in traditional lawyering can be captured in whole or in part in computational originals. By making computational originals the goal for applying the power of computers to the world of financial obligations, we can better avoid the problems inherent in natural language contracting.

We are not alone in this belief. In recent years there has been increasing interest and activity in computational law generally and computational contracting in particular.[22] Prof. Harry Surden, in his ground-breaking 2012 article *Computable Contracts,* describes his vision of a "data-oriented" contract as one "in which the parties have expressed one or more terms or conditions of their agreement in a

[21] Henry Ford apocryphally said, "If I had asked people what they wanted, they would have said faster horses." Kastelle, Tim. "Two Great Innovation Misquotes." The Discipline of Innovation. January 27, 2012. Accessed December 13, 2014. http://timkastelle.org/blog/2012/01/two-great-innovation-misquotes/.

[22] On computational law, see: Love, Nathaniel, and Michael Genesereth. "Computational Law." Stanford Logic Group. June 6, 2005. Accessed December 13, 2014. http://logic.stanford.edu/people/genesereth/papers/computationallaw.pdf. See generally, "Stanford Computational Law." Stanford Computational Law. May 16, 2005. Accessed December 13, 2014. http://complaw.stanford.edu/ ; "Computational Legal Studies." Computational Legal Studies. December 9, 2014. Accessed December 13, 2014. http://computationallegalstudies.com/. On computational contracting, see: Katz, Daniel Martin. "Ethereum Contracts as Legal Contracts - Computational Legal Studies." Computational Legal Studies. June 23, 2014. Accessed December 13, 2014. http://computationallegalstudies.com/2014/06/23/ethereum-contracts-legal-contracts/; Panel Discussion: FutureLaw 2013 - Computational Law and Contracts, Stanford CodeX program, available at http://www.youtube.com/watch?v=KBI8_tv2VDM,

manner designed to be processable by a computer system."[23] He, too, points out limits of natural language contracting, and of attempts to base contract automation on algorithms for parsing natural language originals. Our exercise, set out below, can be viewed as an attempt to thoroughly implement Surden's prescription in a particular case.

This kind of contract automation should also be distinguished from exercises in computer-assisted "document assembly." For some time both commercial and non-profit enterprises have been offering systems that link "expert systems" of guided questions with word-processor based document template libraries to create customized natural language agreements.[24] While such systems can create significant efficiencies for their users, and may be important in near-term efforts to extend access to justice to classically under-represented sectors of society, they are not applying computation to the interior definition and interpretation of the contract.[25] It is still a faster horse.

As introduced above, we also distinguish our exercise from the efforts of some academics and companies to use learning algorithms and other techniques to create automated systems to extract a definitional and logical structure from existing natural language agreement models and then to apply that extracted knowledge to try and emulate their operation. Kingsley Martin is a recognized practitioner in this approach.[26] He has spoken of his desire to "reverse engineer the legal logic" out of a set of documents.[27] While this is a worthwhile exercise, and may be particularly helpful with the

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[23] Surden, Harry. "Computable Contracts." *U.C. Davis L. Rev.* 46: 629-39 (2012).

[24]On the automated management of customized natural language agreements, see: Susskind, Richard E. *The End of Lawyers?: Rethinking the Nature of Legal Services*. Rev. ed. Oxford: Oxford University Press, 2010. Commercial implementations of these approaches include "Exari Intelligent Contract Management Software." Document Assembly & Contract Management Software. Accessed December 13, 2014. http://www.exari.com/About-Us.html, "HotDocs | Document Generation." HotDocs | Document Generation. Accessed December 13, 2014. http://www.hotdocs.com/, and KMStandard's contract automation solutions, "Welcome to." KMStandards. Accessed December 13, 2014. http://kmstandards.com/. CALI's A2J Author is a non-profit example. See "A2J Author | CALI." A2J Author | CALI. Accessed December 13, 2014. http://www.cali.org/content/a2j-author. For discussions of expert systems in legal contexts, see: Engle, Eric Allen. "An Introduction to Artificial Intelligence and Legal Reasoning: Using XTalk to Model the Alien Tort Claims Act and Torture Victim Protection Act." *Rich. J.L. & Tech.* 11, no. 2 (2004). Accessed December 14, 2014. http://law.richmond.edu/jolt/v11i1/article2.pdf; Philip, Leith. "The Rise and Fall of the Legal Expert System." *European Journal of Law and Technology* 1, no. 1 (2010). Accessed December 14, 2014. http://ejlt.org/article/view/14.

[25] Staudt, Ronald W., and Andrew P. Medeiros. "Access to Justice and Technology Clinics: A 4% Solution." *Chicago-Kent College of Law Research Paper No. 2013-41* 88 (2013): 695.Available at SSRN: http://ssrn.com/abstract=2335060

[26] Martin, Kingsley. "Contract Analysis and Contract Standards." Contract Analysis and Contract Standards. March 1, 2012. Accessed December 13, 2014. http://contractanalysis.blogspot.com/. Martin is a principal of KMStandards; see "Company." About. Accessed December 13, 2014. http://kmstandards.com/about.html#Company.

[27] Panel discussion, supra, Note 22.

"legacy" world of existing natural language agreements, our goal is to explore how we might *directly* engineer that logic into computational form. We do admit to having used natural language as our starting point in the project described below, but we think we have done so as part of a fundamentally different exercise from that envisioned by Martin and others taking the reverse engineering approach.

Some researchers have given extensive attention to defining and automating the calculations used in specifying the financial terms of derivatives, swaps and other market instruments, in our terms refining the ability to specify and perform the calculations of function (iii) of financial contracts. This task has attracted the particular attention of financial economists. The efforts of Willi Brammertz and his colleagues are of particular interest here; they propose a formalism for representing the patterns of contingent cash flows that are common to the vast majority of financial contracts.[28] This is important work, but complimentary to the lawyer's task of creating the larger structure of rights and obligations within which such calculations play a part. The automation of that larger structure is our target here.

Economists and transaction parties – including the financial engineers who provide the functional specification for innovative derivatives contracts and structured products – have a language for contingency that is deeply developed but limited in scope. They have evolved a statistical calculus that enables them to analyze the risks and values implied by fluctuations in certain publicly available data, especially the time series of prices of other, "underlying" financial instruments.[29] Their focus on the core functional requirements of the contract is generally a focus on the "happy path" of expected results. This is the chain of events and payments that will flow from the expected deal without the re-directions that arise from contingencies linked to defaults and other "unhappy" events. While the happy path is critically important for the contract, it is far from the only requirement. Contracts, like software, often behave well when confronted with the expected inputs and variables. Bugs and dysfunctions typically crop up in less common use cases, but often occupy much of the specification. The happy path calculations may be booked into the computerized review of transactions at a firm, but frequently without possibilities of the contingency based modifications to the deal that exist in most financial contracts.

Other researchers have sought to outline implementation techniques for embodying contracts into software, including suggesting methods for specifying and manipulating contract terms in purpose-built

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[28] Brammertz, Willi. *Unified Financial Analysis: The Missing Links of Finance*. Chichester, West Sussex, UK: Wiley, 2009.

[29] This is the world of asset pricing, derivatives valuation, and financial risk management. See: Duffie, Darrell. *Dynamic Asset Pricing Theory*. 3rd ed. Princeton, N.J.: Princeton University Press, 2001.; and Hull, John. *Options, Futures, and Other Derivatives*. Ninth ed. Upper Saddle River, N.J.: Pearson Prentice Hall, 2014.

languages.[30] XML is a popular implementation vehicle.[31] These, too, are worthwhile efforts. This paper takes a step back, using the formalism of the deterministic finite automaton to seek confirmation that contracting is a computational process.

There are also efforts to build common taxonomies of financial terms and events that can be used to help specify events and actions in the contract automation process – creating better, and standardized, natural language links between the worlds of performance and computation. One such effort is the EDMC- Financial Industry Business Ontology (FIBO).[32] The linkage of definition to event is a necessary step in creating computable contracts, and one we will discuss more fully below. These efforts can be better understood when viewed in the formal terms which DFA specification requires.

There are historical reasons to temper our expectations for adoption for computational contracting. Half a century ago, there was a flurry of enthusiasm among some legal scholars for the use of abstract logical notation in specifying legal rules.[33] This exercise, with some parallels to our own, never gained much traction. There are reasons to believe, however, that computational expression will provoke wider adoption and application. The restatement in symbolic logic was, for practical purposes, a dead end exercise. It added steps to the drafting process without the potential that computational drafting has for immense eventual gains in efficiency and accuracy through its use in digital processing.

A quarter century ago, a dedicated core of enthusiasts began to explore the topic of Artificial Intelligence and law.[34] This has proved a more sustained effort. The early 1990s saw the organization of The International Association for Artificial Intelligence in Law and the establishment of a dedicated journal on the topic, Artificial Intelligence and Law, both of which persist to this day. The pay-offs

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[30] E.g. Hvitved, Tom. "Contract Formalisation and Modular Implementation of Domain-Specific Languages." January 1, 2012. Accessed December 13, 2014. http://www.diku.dk/hjemmesider/ansatte/hvitved/publications/hvitved12phd.pdf. In addition to its substantive contributions, this dissertation contains a very useful bibliography of prior work on contracts and computation.

[31] Mik, Eliza. "FROM CLAY TABLETS TO AJAX: REPLICATING WRITING AND DOCUMENTS IN INTERNET." *Internet L.* 15, no. 8 (2012). See generally "Legal XML - Workgroups - Substantive - Transcripts Workgroup Website." Legal XML - Workgroups - Substantive - Transcripts Workgroup Website. January 1, 2000. Accessed December 13, 2014. Available at http://www.legalxml.org/workgroups/substantive/transcripts/transcriptwgcharter_2000_03_04.shtml. and "OASIS LegalXML EContracts TC." OASIS. January 1, 2014. Accessed December 13, 2014. https://www.oasis-open.org/committees/tc_home.php?wg_abbrev=legalxml-econtracts.

[32] "EDMC - Financial Industry Business Ontology (FIBO) Foundations (EDMC-FIBO/FND)." EDMC-FIBO FND. Accessed December 13, 2014. http://www.omg.org/spec/EDMC-FIBO/FND/. See, generally Sator, G., M. Biasiotti, and N. Casellas. "Approaches to Legal Ontologies Theories, Domains, Methodologies." 2011 and "Legal Ontology Engineering: Methodologies, Modelling Trends, and the Ontology of Professional Judicial Knowledge." 2011.

[33] Allen, L.E. "Symbolic Logic: A Razor-Edged Tool for Drafting and Interpreting Legal Documents."*Yale Law Journal* 66 (1957): 833-79.

[34] McCarty, L. Thorne. "AI and Law: How to Get There from Here." Rutgers School of Arts and Sciences - Computer Science. March 1, 1990. Accessed December 13, 2014. http://www.cs.rutgers.edu/~mccarty/research/rj90.pdf.

anticipated for this work, however, have yet to fully materialize.[35] One trend in AI and law has been an increasing move in research and application from rule-based approaches (into which our work falls) toward data based approaches, such as the contract analytics described above and, in broader legal contexts, the recent work of companies such as LexMachina and PatentVector and academic work by Daniel Katz.[36] It is our hope that projects such as ours reported here can provide the basis for re-invigorating the rule-based approaches, with the potential for more powerful and transformational application in financial contracting.[37]

This is a more propitious time for pursuing computation and law.[38] The impacts of technology more generally in society suggest that there is nothing all that special about law other than a deeply entrenched systemic conservatism. In a brief piece posted in 2008, Kevin Kelly posited the useful notion of a field being "Turing'd:"[39]

> I had an epiphany recently on why some varieties of professionals are more welcoming of disruptive technology than others. I realized the types of pros who are most eager to employ the latest technology are those fields which have already been Turing'd. We have this long list of tasks and occupations that we humans believe only humans can do. Used to be things like using tools, language, painting, playing chess. Now, one by one they get Turing'd. A computer beats

[35] See: "A History of AI and Law in 50 Papers: 25 Years of the International Conference on AI and Law Artificial Intelligence and Law." *Artificial Intelligence and Law* 20, no. 3: 215-319 (2012).available at https://cgi.csc.liv.ac.uk/~tbc/publications/ICAIL25AuthorsVersion.pdf: and Leith, P. "The Rise and Fall of the Legal Expert System." *European Journal of Law and Technology*1, no. 1 (2010), available at http://ejlt.org/article/view/14/1.

[36] For the vendor approaches, see "Legal Analytics by Lex Machina." Lex Machina. Accessed December 13, 2014. https://lexmachina.com/ and Patent Vector, http://www.patentvector.com/ (password access may be required). On the academic side, see Katz, Daniel Martin. "Quantitative Legal Prediction – or – How I Learned to Stop Worrying and Start Preparing for the Data Driven Future of the Legal Services Industry." *Emory Law Journal* 62: 909-966 (2012), available at http://ssrn.com/abstract=2187752. See generally "Using Data to Predict Supreme Court's Decisions." Daniel Martin Katz - Michigan State University. Accessed December 13, 2014. http://www.katz.law.msu.edu/.

[37] On rule-based approaches, see Engle, Eric Allen. "An Introduction to Artificial Intelligence and Legal Reasoning: Using XTalk to Model the Alien Tort Claims Act and Torture Victim Protection Act." *Rich. J.L. & Tech.* 11 (2004). Available at http://law.richmond.edu/jolt/v11i1/article2.pdf. More generally, see Lettieri, N., and S. Faro. "Computational Social Science and Its Potential Impact upon Law."*European Journal of Law and Technology* 3, no. 3 (2012). Available at http://ejlt.org/article/view/175/267. On their application in agreements, see Ossowski, Sascha. *Agreement Technologies*. Dordrecht: Springer Science Business Media, 2013.

[38] E.g. Hodson, Hall. "AI Gets Involved with the Law." *The New Scientist*, 2013. Available at http://www.newscientist.com/article/mg21829175.900-ai-gets-involved-with-the-law.html.

[39] Kelley, Kevin. "The Technium: Turing'd." The Technium: Turing'd. March 21, 2008. Accessed December 13, 2014. http://kk.org/thetechnium/2008/03/turingd/.

them. Does it better. So far we've [sic] can check off arithmetic, spelling, flying planes, playing chess, wiring chips, scheduling tasks, welding, etc. All have been Turing'd.

Once the field has been Turing'd, additional computerization becomes much easier to implement. Law and finance are arguably there now, in practice if not yet in academia. The role of technology in many aspects of legal practice is fully established.[40] John McGinnis and Russell Pierce are hailing the application of "machine intelligence" as a coming "great disruption" in the role of lawyers in the delivery of legal services.[41] We also note that the automation of accounting, analysis and trade execution is widely implemented in the operations of companies active in the financial markets.[42] In the context of these existing applications and the recognized need for better understanding of contractual relationships in the wake of the 2007-09 financial meltdown, the contractual automation we envision is as much an opportunity for improvement in contractual modeling, specification, and measurement, as it is a threat to status quo processes and traditions.

IV. Describing a Financial Contract as a Deterministic Finite Automaton

The central contribution of this paper is to demonstrate, through a detailed example, that *a well designed financial contract is a state-transition system*. Moreover, we can structure such a financial contract as a particular sort of state-transition system, namely a deterministic, finite automaton (DFA).

A financial contract is a definition of a relationship between counterparties. The contract specifies the contingencies that can affect the relationship, and how the relationship should react to them. This reaction is organized around a set of mutually exclusive states – e.g., "performing," "in default for missed payment," etc. – that describe the status of the relationship at any point in its life. By agreeing to a contract, the counterparties commit, with provisions for enforcement, to adhere to the contract's prescribed rules of behavior. These commitments are a powerful device for coordinating financial activity, because they alleviate much of the "social uncertainty" inherent in economic interactions. For example, a banker who issues callable debt to fund a portfolio of (prepayable) mortgages is at greater liberty to lend, knowing that she can rely on her investors to buy back their debt, should the mortgage

[40] See generally LTN Law Technology News. Accessed December 13, 2014. http://www.lawtechnologynews.com; LegalITInsider. Accessed December 13, 2014. http://www.legaltechnology.com/; and "Exhibitors and Events at LegalTech 2014." LegalTech. Accessed December 13, 2014. http://www.legaltechshow.com/r5/cob_page.asp?category_id=76594&initial_file=cob_page-ltech.asp.

[41] McGinnis, John O., and Russell G. Pearce. "The Great Disruption: How Machine Intelligence Will Transform the Role of Lawyers in the Delivery of Legal Services." *Fordham Law Review* 82, no. 6: 3042-066 (2014). Available at http://fordhamlawreview.org/assets/pdfs/Vol_82/No_6/McGinnisPearce_May.pdf.

[42] See the articles and associated references in Brose, Margarita S., Mark D. Flood, Dilip Krishna, and Bill Nichols. *Handbook of Financial Data and Risk Information I Principles and Context*. Vol. I. Cambridge: Cambridge University Press, 2013., and Vol. II: Software and Data.

portfolio prepay unexpectedly. Without such a guarantee of her investors' behavior, her own lending behavior would be more circumspect. Financial contracts are thus an elemental catalyst for the division of labor and exploitation of comparative advantage.

The DFA structure of a financial contract is not metaphorical. A DFA is a mathematical formalism with precise requirements, which a contract may or may not meet. We argue that a "well designed" financial contract is one that satisfies these requirements, and thereby gains access to the substantial body of formal results elaborated for DFAs in the theory of computation.[43] Conversely, it is possible that a given financial contract, "observed in the field," will fall short of the requirements of the DFA formalism. Such deviations from the ideal represent design compromises, and the distance between the actual and the ideal in any given case might form the basis for an assessment of the quality of the legal craftsmanship involved.

We have allowed ourselves two elements of simplification in the subsequent discussion. First, our example – a "toy" loan contract – is intentionally streamlined to avoid an excess of contractual provisions beyond the basics needed to illustrate the core of our argument and demonstrate the feasibility of the approach. We initially targeted some fully formed, practical examples of financial transactions, including the International Swap Dealers Association (ISDA) 2002 ISDA Master Agreement and the 1997 International Foreign Exchange Master Agreement (IFEMA).[44] In both cases, we were able to make progress in mapping the agreements into the DFA formalism, but both agreements are crowded with a profusion of particulars that add significantly to the modeling burden while distracting attention from the central concepts. We leave these more ambitious mappings as projects for future research.

Second, and more fundamentally, we have chosen the most basic computational model available, the deterministic finite automaton (DFA), as the formalism to represent the structure of our contract. Computation theory has accumulated a modest bestiary of different computation models, appropriate for different sorts of tasks, such as representing system behavior or parsing computer source code or natural language documents that conform to a specific grammar. One factor that distinguishes these models is their level of expressiveness, meaning the degree of intricacy of the structure that they can capture. The fact that the DFA computational model is sufficiently expressive to represent our "toy" suggests that contract law and drafting have evolved to embody computational logic at this relatively simple level of sophistication for managing actual relationships.[45] The agreed relationship must

[43] There are numerous excellent textbook treatments of DFAs and the theory of computation, including: Sipser (2006), supra; Kozen (1997), supra; and Rosenberg (2010), supra.

[44] The ISDA agreement covers over-the-counter swap transactions; the agreement is available at: "ISDA Master Agreement." ISDA Bookstore. http://www.isda.org/publications/isdamasteragrmnt.aspx . The Foreign Exchange Committee manages the IFEMA; the agreement is available at "IFEMA Agreement." Federal Reserve Bank of New York. http://www.newyorkfed.org/fmlg/ifema.pdf.

[45] There are several alternative representations with expressiveness equivalent to the DFA. We discuss tabular, graphical, and regular expression presentations below. In addition, any DFA can be converted to an equivalent nondeterministic finite automaton (NFA) – and vice versa – via a straightforward programmatic exercise. The NFA representation adds a layer of abstraction, but is typically more compact than the corresponding DFA; see Sipser

ultimately be interpreted the counterparties, and potentially by the courts or other arbitrators, in a broader context that may have changed substantially over the course of the relationship. Clarity and simplicity of the contract are important virtues; ambiguity and sophistication are not. We return to this fundamental point in the conclusions.

Table 2: A "Toy" Loan Agreement

The Formalism and the Contract

We propose to represent a financial contract as a discrete finite automaton. This is an exercise in parallel specification. The first specification is the "toy" contract, embodied in legalese, shown in Table 2 . The second specification is a DFA representation, in the tables and figure below, of the same structure of rights, obligations, actions and contingencies.

Formally, the DFA specifies a computation as a 5-tuple, $(Q, \Sigma, \delta, q_0, F)$, consisting of:[46]

1. a finite set of states, denoted Q
2. a finite set of input symbols (events) called the "alphabet" (Σ)
3. a transition function ($\delta : Q \times \Sigma \rightarrow Q$)
4. a start state ($q_0 \in Q$)
5. a set of accept (end) states ($F \subseteq Q$)

The core idea is that the contract defines a finite, mutually exclusive, and exhaustive collection of states that can describe the possible conditions of the relationship between the counterparties. At any point in the life of the agreement, the relationship is in exactly one state, $q \in Q$. Each state ($q \in Q$) represents a kind of "landing place", like the squares on a chessboard. Which particular state is operative at a given point will depend on what has occurred, i.e., the sequence of observed events, $e_1, e_2, \ldots \in \Sigma$ up until that point. Organizing a contractual relationship around discrete states, such as "performing", "cancelled", or "defaulted," is what good drafters of agreements do, even when this discipline is more instinctive than formal.

The alphabet represents the discrete set of inputs (Σ) that the agreement recognizes.[47] For example, these might correspond to information events, or actions taken by the counterparties. Not all events

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(2006), section 1.2. More expressive computational models include pushdown automata (Sipser, 2006, at 109 et seq) and Turing machines (Sipser, 2006, at 137 et seq). Computer languages, such as Java or C++, are often described as being "Turing complete," meaning that they have the full expressive power of a Turing machine.

[46] We follow Sipser's (2006) notation here.

[47] For those unfamiliar with computational theory, the labels of "states" and "alphabets" may seem arbitrary or even counterintuitive given their conventional meaning; we ask that you suspend the objection and simply understand their use in this context. Such a step should not be too hard for lawyers and economists, as both professions are full of jargon that defies understanding in conventional terms. Use of the label, "alphabet," to describe the space of possible inputs derives from the commonplace use of automata to model parsers for

are relevant to an agreement. Our "toy" loan agreement does not countenance the outcome of the World Series or fluctuations in the spot price of crude oil, for example. An important function of a contract is to make positive commitments to address those specific events and actions that rise to the threshold of relevance for the relationship – the alphabet, Σ, in DFA terms. Moreover, the contract provides the important simplification of discretizing the infinite gradations of possibility that might describe the real world into a finite set of events as measured by the contract. For example, changes in creditworthiness under the ISDA master agreement are measured not by the gradual deterioration of an entity's credit quality, but by the occurrence of one of several discrete credit events, such as "failure to pay" or "repudiation," as announced by the appropriate ISDA determination committee. The resulting finiteness of the event space has important implications for the possible complexity of the agreement. Formalizing these inputs as the alphabet Σ helps to clarify these principles.

The transition function describes how the state of the relationship will change in response to the arrival or announcement of particular events. Using a board game metaphor, the transition function describes how the relationship should move from one state (or space on the game board) to another, based on a given event outcome (such as a roll of the die in the board game). That is, with the relationship starting in state $q_1 \in Q$, the transition function defines a mapping $q_1 \times e \rightarrow q_2$ that describes that the state of the relationship should change from state q_1 to q_2 in response to the arrival of event $e \in \Sigma$. In principle, any combination of initial states and observed events is conceivable, but in practice, an agreement will simply ignore some events in certain states. In other words, the response to certain state-event pairs is to remain in the initial state, or $q_1 \times e \rightarrow q_1$. For example, if the contract is in the "cancelled" state, then the occurrence of a "declaration of bankruptcy" event will not change the state; the agreement remains cancelled: $q_{\text{cancelled}} \times e_{\text{bankruptcy}} \rightarrow q_{\text{cancelled}}$.

This process of event and transition goes forward until the automaton reaches an end, or "accept," state.[48] The start state, q_0, is the landing spot where the relationship begins, and an accept state, $q_T \in F$, is a spot where, if reached, the process ends. The sets of Q and Σ are both finite (hence the "F" in DFA), and the transition function is deterministic in that it always has one and only one output of change or remain (hence the "D"). We assert that contracts in general, and financial contracts in particular, should follow a similar kind of step by step logic, matching information against a current state of facts and contract execution specifications to lead to a next state. Put even more strongly, *we believe that natural language financial contracts are (or should be) themselves computational automata in this sense*. That is, as the legal machinery for describing how a relationship will evolve in response to key events, a financial contract adheres to the prerequisites of finite automata from computational theory. If this is so, a series of applications and conclusions, many already well-established in computational theory, may become available, with potentially far-reaching implications.

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programming languages. In this application domain, the language compiler sees a program arriving as a sequential string of characters from a restricted universe, such as the ASCII character set.

[48] The nomenclature, "accept," also comes from the tradition of parsing computer languages, where the parser is looking for certain strings of characters, such as "X=5;" or "PRINT X;" that it understands or "accepts."

V. Representations of the Target as a DFA

To test this proposition, we have sought to describe a financial contract fully and explicitly in the terms of a DFA. To keep this initial example focused and manageable, we apply the method to the understanding embodied in the "toy" agreement – the loan arrangement set out in Table 2. We have sought, within its simplified context, to have at least one example of most of the common moving parts described in Table 1. The "toy" agreement specifies one loan and only two repayments. The interest is an easy percentage (5% non-compounding) and is specified as an explicit dollar amount in the repayment itself, thus obviating the need for an ancillary interest rate calculation process.[49] There is one warranty, one covenant, and one other outside, non-payment related event that can trigger default. The default process has a single set of time frames and notice specifications, and the acceleration, if triggered, is still for the sum certain of the outstanding payments, without additional penalty. We have not provided for collateral or a guaranty. Notwithstanding these simplifications, we believe our "toy" provides a reasonable proof of concept for the possibility of describing a financial contract in the basic computational representation of a DFA.

There are at least three standard means for representing the "toy" contract in explicit computational form as a DFA. These three are equivalent in the sense that they all capture the same information, and there are standard procedures for translating (without loss of information) among the three: (i) a visual plot of the state-transition network, (ii) a tabular (or matrix) listing of the elements of the 5-tuple, and (iii) a so-called "regular expression."

Graphical Representation

The first version we present of the DFA for our "toy" agreement, set out in Figure 1, is a *graphical representation* of the state-transition network. The rounded bubbles represent the different states in the DFA. The labels on the arrows describe elements from the event alphabet that effect transitions from one state to another. The arrows themselves show the effect of the transition function applied to the state and alphabet elements. The start state is represented by the solid bubble at the top, and the three accept states of contract fulfillment, cancelation, and litigation appear as bubbles with double borders at the bottom. The "happy path" through the relationship is indicated by the green bubbles, and the sequence of bold green arrows that connect them.

Figure 1: Graphical representation of the DFA for the "toy" contract

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[49] Of course, interest calculations are themselves computable and therefore could have been modeled within the context of the DFA here. However, modeling these details (which, in some cases, can be quite complex indeed) would likely involve a significant digression into questions with little interesting legal content, other than the resulting final number. It is commonplace in modeling system state to encapsulate this sort of modular functionality, and delegate it to a distinct object or function dedicated to that task. The hard-coding of the bottom-line payment amounts ($525, $550 and $1075) has the same simplifying effect.

The representation in Figure 1 is a partial simplification, because it omits many of the transitions in the interest of clarity. Most events in the alphabet, most of the time, leave the state unchanged; the events are irrelevancies in the context of those states. To be complete, the graph should include transitions for all such events, looping back to the same state. In addition, a few alphabet items, such as those relating to litigation or cancelation, are relevant in every state, but always transition to the same terminal state. We have omitted these repetitive arrows from Figure 1 to avoid creating a hairball of visual confusion. These repetitive elements do, however appear in the tabular presentation of the DFA to follow. Note too that an input event that represents an agreed waiver or amendment will necessarily result in a new DFA representation with a different set of elements. In the context of the current DFA, these events provoke a transition directly to the terminal "cancelation" state.

Tabular Representation

The next version is a tabular specification of the elements in the 5-tuple: $\{Q, \Sigma, \delta, q_0, F\}$. Recall that for each state $q \in Q$, most events leave the state unchanged – the input is irrelevant to the next move from that state and no change is made. Also, for any state, the input $T \in \Sigma$ (the final row of Table 4) that represents a waiver, amendment, or agreed termination of the agreement has occurred takes the logic right to an accept state of termination of *this* agreement/DFA. In the case of a waiver or amendment, the change provoked by these events will result in a new DFA with a different set of elements. For ease of comparison, we also list a cross-reference to the section in the natural language version to which the state or alphabet most relates.

The tabular presentation emphasizes an important point about DFA formalism: the two spaces of states (Q) and events (Σ) are both simple sets. That is, there is no ordering or ranking among the elements of either set; any reshuffling of the rows of Table 3 or Table 4 would have no impact on the DFA. The only special status is given to start state, q_0, and the accept states, F, declared as separate members of the 5-tuple. The subtle implication of the lack of ordering among the elements of Q and Σ is that the states of the DFA are "memoryless." In technical jargon, each state exhibits the Myhill-Nerode property of being independent of its past.[50] We humans may attend to the narrative of events that brought the relationship to a particular state, but the DFA does not care. Once a state is reached, the history that led there is irrelevant; all that matters is the one-step-ahead process of responding to whichever next event may arrive. In other words, any relevant history that brought the relationship to a particular state is encapsulated in the fact of being in that state. As Rosenberg (2010, p. 56) puts it, "the state of a system comprises that fragment of its history that allows it to behave correctly in the future." This requirement of memorylessness inherent in the DFA formalism implies a discipline that should govern the drafters of financial agreements. In a nutshell, it restricts the contract to consider only "regular" sequences of

[50] Students of statistics will recognize this as the first-order Markov property, and should note that a first-order Markov chain is the probabilistic counterpart of a finite automaton. In particular, the same characteristic memorylessness of a Markov chain also obtains here, even though the DFA is a non-random process. For a detailed discussion of the Myhill-Nerode property and its important implications, see Rosenberg (2010), esp. ch. 4-5.

events, as described below. It also has powerful implications for the computational complexity of the contractual machinery.

Table 3: Contract States (Q)

Table 4: Event Alphabet (Σ)

Table 5: Transition Function (δ)

Just as we simplified the graphical representation of the DFA in Figure 1, we abbreviate the transition function in Table 5 by suppressing the "stay-in-place" transitions that return the system to the same state in response to an event. These event occurrences are "irrelevant" to a given state if a self-transition – i.e., ignoring the event, rather than provoking a rejection – is an appropriate response.[51] Unlike a parser, for example, which has a responsibility to reject character sequences that are unacceptable, a financial agreement has the flexibility simply to ignore most of the superfluous event occurrences. This creates a proliferation of self-transitions that would otherwise clutter Table 5 with relatively trivial entries. In other words, the application context for financial agreements is less tightly controlled than for programming-language parsers, and contracts should be relatively permissive and robust to irrelevant event occurrences. Table 5 also suppresses "omnipresent" events such as the filing of a lawsuit (event "D") that could occur – and be relevant – in any state. For completeness, we also present in Table 6 the full transition function as a matrix, where the rows correspond to the states in Table 3, the columns correspond to the event alphabet in Table 4, and the state listed in in each cell is the result of the transition function applied to the combination of the initial state (identified by the row) and event (column) values. Unlike Table 5, the full transition function in Table 6 does not suppress "irrelevant" and "omnipresent" events. To condense the matrix to a single page, Table 6 does, however, omit the natural language queries and correlates included in Table 3 and Table 4.

Table 6: Full Transition Matrix

Regular Expression Representation

[51] Knowing when to ignore events, rather than reject (for example, by triggering a back-office investigation), is an important judgment call for contract drafters and implementers. For example, expiration of the statute of limitations with respect to the obligations of the borrower (event "E" in Table 4) before delivery of the principal (event "F") is a physical impossibility, so there is little point in developing error-handling procedures for this case. On the other hand, multiple occurrences of event "F" in quick succession would be a plausible clerical error, and a rejection procedure might be appropriate to handle this case.

Our final representation of the DFA is a so-called "regular expression." A primary upshot of a finite automaton is a declaration of the set of all strings – concatenated sequences from the event alphabet – that the machine will accept. These are the event sequences for which the contract has scripted some appropriate behavior of the counterparties, and which leave the automaton in one of its "accept" states. For (a highly simplistic) example, a given contract might accept a happy-path event sequence of "sign"-"pay"-"quit," meaning sign the deal, then make the promised payments, then terminate the relationship. In contrast, a sequence of "quit"-"pay"-"sign" would be nonsensical, and the contract's DFA should declare its inability to process this sequence of events. In the case of our "toy" contract, we can see that the event sequence "**ACFGQPR**" defines the happy path, while the shortest event sequence resulting in a final state is "**AB**."

Extending the metaphor of an alphabet of events, these collections of all acceptable (by the automaton) event sequences are called the "language" recognized by the DFA. In short, the DFA defines a "grammar" for the language. As noted above, a DFA is one of the simplest computational formalisms, and therefore supports some of the least expressive languages, known as the "regular" languages. A useful feature of the regular languages is the existence of a shorthand notation that can capture an entire regular language with a single snippet of event-sequence patterns, known as a "regular expression."[52]

The regular expression shorthand that describes a particular regular language is stated entirely in terms of the characters from the event alphabet, perhaps interspersed with a few control characters and wildcards that are special to the regular expression notation itself. The DFA that governs the regular expression is implicit rather than explicit. There are programmatic techniques for generating regular expressions from DFAs. However, just as "cannot" "can not" and "can't" are all character strings expressing the same concept, a given DFA can have many regular expressions. Fortunately, it is usually easy to justify that the shortest regular expression for a given language is the "best" one to use, and it is possible for the programmatic generators to produce this unique, optimal version. Perhaps more surprisingly, there are programmatic techniques to move in the other direction, and recover the DFA implicit behind the regular expression for a language. Here again, the implicit DFA is not unique, but there are rules for choosing the "best" from among the many equivalent possibilities.[53] The result of all this is a set of tools for converting our DFA into a regular expression representation and back again.

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[52] A familiar, if trivial, example of a regular expression is the use of an asterisk to create a wildcard query, such as using "*.txt" to search for all text files in a file system. This regular expression accepts any filename with the four characters ".txt" at the end. There are a number of competing standard versions of the shorthand notation for regular expressions; see Friedl, Jeffrey E. F. *Mastering Regular Expressions*. 3rd ed. Sebastopol, CA: O'Reilly, 2006., at: http://regex.info/book.html. For a more technical introduction, see ch. 10 Aho, Alfred V., and Jeffrey D. Ullman. "10." In *Foundations of Computer Science*. New York: Computer Science Press, 1995, at: http://infolab.stanford.edu/~ullman/focs.html.

[53] For an introduction to the translation between DFAs and regular expressions, see Sipser (2006), section 1.3. For a deeper discussion, see Rosenberg (2010), especially section 5.2.

We use the method of state elimination to convert the DFA represented in Figure 1 into an equivalent regular expression shorthand for the set of event sequences the DFA accepts. The key operation in state elimination is to replace the event arrows in the DFA with arrows describing event sequences, while still preserving the state-transition logic of the full graph. For example, consider this snippet from Figure 1:

{***Pmt. 1 due***} –[**B**]–> {***Default (borr.) $ miss***.} –[**K**]–> {***Borrower notified of payment default***}

One can eliminate the central state, "Default (borr.) $ miss." without disrupting the overall state-transition logic by replacing the elided state with at joint transition arrow, labeled as an event sequence:

{***Pmt. 1 due***} –[**BK**]–> {***Borrower notified of payment default***}

This is a particularly simple example state elimination, but the procedure extends in a straightforward way to more involved states in the network.[54] The regular expression shorthand has a few syntactic tools to allow it to express some common DFA structures with compact notation. We use a basic version of the Unix notation here. There are numerous competing shorthand notations for regular expressions. For example, Hopcroft, et al., use the "+" symbol to indicate a "union" or branching pattern, where the path can follow either one of several branches. Sisler uses the "∪" symbol for the same purpose.[55]

- Parentheses indicate association, meaning that the enclosed regular expression should be evaluated first, before considering the rest of the pattern.
- The "*****" wildcard indicates that the preceding event (or parenthetical pattern) can appear any number of times, including zero times.
- The "**?**" wildcard indicates that the preceding event (or parenthetical pattern) can appear zero or one time only.
- A vertical bar between two states, such as **A|B**, indicates that the path can follow either event **A** or event **B**.
- A concatenation of states enclosed in square brackets, **[ABC]**, indicates that the path can follow any of those events. That is, **[ABC]** is logically identical to: **A|B|C**.

For example, the "toy" contract depicted in Figure 1 has several pathways from the start state that lead to a relatively "rapid demise" of the relationship: **AB** or **ACBE** or **ACBD**. We can capture all three in a single regular expression, **A(B|CB[ED])**. The happy path for the "toy" contract is given by the event sequence: **ACFGQPR**.

Note that the regular expression for a given DFA is not unique. For example, it is easy to see that the two expressions, **A(B|CB[ED])** and **(AB|ACB[ED])**, recognize the identical event sequences and

[54] For a more detailed introduction to the state-elimination method, see Hopcroft, et al. (2001) section 3.2; or Sipser (2006), pp. 66-76. Hopcroft, et al. (2001) describe two general methods for the DFA-to-regular expressions conversion, namely path induction and state elimination. The two methods are equivalent in the sense of producing equivalent regular expressions.

[55] See Hopcroft, et al. (2001), section 3.3, for an overview of the Unix syntax for regular expressions.

are therefore functionally equivalent. We find it convenient to organize the regular expression for our "toy" contract as the union of four key segments, corresponding to: (a) "rapid-demise" paths; (b) the "happy" path; (c) "unhappy" paths (payment and non-payment defaults) around the first payment date; and (d) "unhappy" paths around the second payment. Eliding the derivation of these four expressions, the overall regular expression representation of the DFA is:[56]

A(B|CB[ED])|	*Rapid demise*
ACF(G(BK)?)QPR|	*Happy path*
ACF([HIJ]LN)*(GBK|[HIJ]L)O(S|B[DES])|	*Unhappy* 1
ACF(G(BK)?)Q([HIJ]LN)*(PBK|[HIJ]L)O(R|B[RED])	*Unhappy* 2

Note that the final segment, "unhappy 2," follows the "happy path" up to state **Q** (payment 2 accruing) and then diverges into the various ramifications of payment and non-payment default from that state. The "happy path" segment here includes a wildcard sub-segment, **(BK)?**, covering the possibility of a missed payment that is quickly cured. Similarly, the two "unhappy" segments include a wildcard sub-segment, **([HIJ]LN)***, indicating that the contract can tolerate an arbitrary number (including zero), *****, of any of the three non-payment defaults, **[HIJ]**, as long as they are cured in a timely fashion, **LN**.

At first glance, the regular expression representation of the contract may seem like a hopeless bit of gobbledygook, but, like a sort of contractual DNA, it is actually a powerfully compact distillation of two full pages of legalese. In addition to its theoretical benefits, regular expressions are also enormously practical, as users of the venerable Unix command-line utility, grep (the "global regular expression printer"), can testify. In particular, the regular expression's string of symbols provides a simple and intuitive measure of the "complexity" of the contract, namely the length of the string. One might object that, because a DFA's state transition-network and its regular expression are generally not unique, the complexity score is arbitrary. This objection is ill-founded, however, because there are programmatic techniques to reduce any DFA to a theoretical minimum state-transition network, and standard techniques for representing any given DFA as a regular expression. The complexity score should measure the length of a standardized regular expression for the minimized DFA. In the example here, the complexity score is 109.

A contract should be as simple as possible, but no simpler. Note that the bulk of the contract's complexity (75.2%, to be precise) arises in the two nexuses of "unhappy" ramifications. The two unhappy substrings account for 82 symbols in total, or 82/109 = 75.23% of the overall string length. Unsurprisingly, much of the hard work of managing a relationship occurs not when things are going well,

[56] The derivation of the regular expression from the state-transition network via state elimination is mildly tedious, but a useful exercise for deepening one's understanding of the DFA machinery. Conversely, it is also instructive to follow each of the four segments of the regular expression here as a set of directions through the "maze" of the state-transition graph in Figure 1, to verify that the regular expression describes all possible pathways through the network.

but rather when the process starts to deviate from the happy path. Much of the value of good contracts and good lawyering derives from the seemingly tedious planning for all the ways that a relationship might run off the rails.

Note too that financial risk and valuation models – the core of financial engineering – tend to ignore the complications of the "unhappy" nexuses, focusing instead on probabilistic models of the happy path. Implicitly, this seeming negligence relies on an assumption that all unhappy relationships are idiosyncratic, so that the manager of a well-diversified investment portfolio can safely ignore these high-maintenance details. In many cases, such as bank commercial loan portfolios, the lumpiness of the holdings – a relative handful of large, specialized relationships – denies the portfolio manager the luxury of simple diversification. Alternative mechanisms, such as securitization and syndication have evolved to spread the risks in these situations. Formal modeling of these complicated portions of financial relationships as DFAs may make them more measurable, and therefore more manageable. If so, such modeling has the potential to create significant value by reducing overall financial drag.

VI Discussion and Directions for Further Exploration

The foregoing provides evidence for our proposition that the structures embodied in financial contracts are state-transition systems. The key is in recognizing that the state-transition structure is sufficiently fundamental to a financial agreement that we can represent it using the standard computational formalism of a deterministic finite automaton without disrupting the contract's organizing principles. If this is true, why does it matter? By identifying the DFA that undergirds a contract, we expose the entire edifice to the tools and techniques developed in the computational and linguistics communities to work on finite automata.

For our "toy" contract, the preceding Section presents three canonical representations – graphical, tabular, and "regular" – of the underlying DFA. Taken together, they are equally valid embodiments of the process set out in natural language in the "toy" contract, with the added benefit of being expressly computable. By this we do not mean that the DFA restates the natural language contract; rather, each is a valid embodiment of a logic that exists as an independent algorithm waiting, as it were, for description.

This possibility for multiple forms of expression resembles the way that an algebraic proposition can be described both as a word problem and as a numeric formulation. And as with algebra, representing the proposition in the formal language of mathematics opens the proposition to a number of tests, applications and manipulations that are much more difficult to access when it remains a word problem described in natural language. For example, it is possible to craft the three representations in such a way that they are precisely formally equivalent. A key to establishing this mutual equivalence lies in the application of techniques for minimizing the finite automaton. One of the contributions of the Myhill-Nerode Theorem is that there exists a *unique* smallest finite automaton that will accept a given "language" of event sequences defined by the regular expression.

We did not apply this sort of formal minimization logic in the construction of the "toy" DFA, preferring to maximize instead the common-sense legal semantics of the DFA. It will be instructive to see how far our legally optimized presentation is from the theoretical minimum. This gap is likely to widen as we apply the approach to more realistic agreements. It is important to recognize that legal contracts are ultimately devices for coordinating human activity, and much of their effectiveness derives from their enforceability. The lender is willing to relinquish the principal, in part because she knows authorities exist to enforce repayment if necessary. These authorities involve human interpreters – judges, juries, arbitrators, etc. – who must be able to parse the agreement in the crucial tasks of dispute resolution. Else, the contract has failed to meet one of its most important requirements.[57]

We started with the natural-language representation (old lawyering habits die hard), and retrofitted the computational model. The process of representing the state-transition logic, however, clarified requirements and assumptions, leading us in turn to amend the natural language version. The final "toy" contract is thus the outcome of iterative approximations to both the natural language and the DFA, so that neither form has full precedence. Indeed, creating the DFA helped us to refine the logic of the transaction as a whole, and to clarify the role of standard elements as warranties and defaults. A natural next step will be to perform a similar exercise on a more ambitious agreement, perhaps a standard master agreement taken from the realm of actual financial practice. We can also envision a process of building a useful master agreement of our own in computational terms from the ground-up. Perhaps the next iteration of ISDA will be drafted in just such a fashion.

As we pursue that direction in our research, one interim goal will be to assemble some of the existing tooling – much of it described in textbooks and existing software packages – into a suite of functionality to make these transformations as a set of programmatic, push-button tasks.[58] This toolkit will be instrumental as we begin to apply the state-transition approach to more realistic contracts.

As the analysis moves to more complex and realistic examples, we will want to expand the toolkit beyond DFAs to include nondeterministic finite automata (NFAs). NFAs are a more sophisticated class of automata that allow multiple alternate transitions from a state in response to an event.[59] This is a

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[57] One can imagine a science-fiction future in which contract enforcement is itself fully automated and delegated to machines for efficiency's sake. The ethical ramifications of such an institutional arrangement could be very extensive.

[58] Examples of relevant software packages include Berkeley YACC, available at: "BYACC – Berkeley Yacc – Generate LALR(1) Parsers." BYACC – Berkeley Yacc – Generate LALR(1) Parsers. Accessed December 13, 2014. http://invisible-island.net/byacc/byacc.html; and GNU Bison, available at: "Bison - GNU Project - Free Software Foundation." Bison - GNU Project - Free Software Foundation. Accessed December 13, 2014. https://www.gnu.org/software/bison/. Textbook sketches of many of the programmatic transformations are available in, for example, Sipser (2006), Rosenberg (2010), and Aho, Alfred V., Monica S. Lam, Ravi Sethi, and Jeffrey D. Ullman. *Compilers: Principles, Techniques, & Tools*. 2nd ed. Boston: Pearson/Addison Wesley, 2007.

[59] Multiple responses to the same event appear to be a contradiction in terms. However, the multiple transitions are not meant to be taken literally. Instead this is a modeling abstraction that still produces the same final

representational convenience that affords significant simplification of the state-transition graph in many cases. Note that NFAs are semantically equivalent to DFAs. They support the same set of regular grammars, and there exist standard techniques for translating between DFA and NFA representations. Any state-transition system with a DFA representation can be converted to an equivalent (in terms of its acceptable sequences of events) NFA and vice versa. It is already clear from our preliminary forays into standard financial master agreements that this sort of flexibility will be very useful.

The DFA captures the central legal logic of the contract. However, the DFA *per se* does not capture the entirety of the agreement. There are, in addition, two import semantic conversions that round out the picture. First, a "measurement" or "feature extraction" process is defined by the agreement to convert from the salient events and occurrences in the real world to the tidy, finite microcosm of the DFA. For example, the borrower represents that his assets exceed his liabilities under generally accepted accounting principles. The DFA requires this simple determination – yes or no – whether the borrower is solvent, encoding this fact as an alphabet element. The measurement process to reach this decision will typically involve a multitude of assumptions, interpretations and judgments of accounting, but the "bottom line" will have the full clarity of a discrete, binary variable. Requiring the counterparties to maintain this clarity is a valuable function of the contract. The translation of external occurrences into the event alphabet is still mostly handled by the natural-language definitions in the contract. The details and nuances of this measurement task have been excluded from the scope of the present paper, but they are an important area for follow-on research. The work on ontologies described above is one example of some of the steps this process.

Second, there is a question of the semantic interpretation of the states and transitions in the automaton. For example, when the automaton is in state "xL", this fact about the DFA has important implications for the parties back in the real world; one of the parties is likely to be filing litigation in a specified jurisdiction, requiring documentation of claims, etc. Some interpretative mechanism is required to understand that all of these messy contextual details are entailed by the simple marker, "xL". This explicit mapping from the DFA to the external legal context is a sort of formal semantics that requires additional attention in subsequent research.[60] Some of the old challenges – and perhaps advantages – of ambiguity and lawyer brains creep back into the process through the use of natural language queries as the basis for specifying an alphabet triggering event.

As an aside, we note that one standard technique for building state-transition systems is to focus on the entity-specific states that might describe each participant individually in a "system" of interacting entities, such as contractual counterparties. The state of the relationship is then assembled from all the

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behavior in the contract (or other system), but that can typically be expressed more concisely. See Sipser (2006, ch. 1) for further discussion.

[60] For an introduction to computational logics and formal semantics, see Huth, Michael, and Mark Ryan. *Logic in Computer Science: Modelling and Reasoning about Systems*. 2nd ed. Cambridge, U.K.: Cambridge University Press, 2004.

possible combinations of the individual states. This is called a "product automaton."[61] For example, each spouse in a marriage might individually be in the state "happy" (H) or "unhappy" (U), so that the state of the marriage is described by one of the product states: HH, UH, HU, or UU. The work we present here suggests that the law does not work this way. That is, a financial contract does not attempt to describe the state of the counterparties directly, and then combine these counterparty-specific states into their various interactions. Rather, the contract reifies the relationship as a distinct thing. The states and transitions between states that structure the contract then describe the relationship object per se. Abstractly, one might instead establish a structural equivalence between the states of the relationship and those of a product automaton; for example, one might re-label the product states suggested above to say that the marital relationship is in a "successful" state if the product state of the spouses is HH. As a practical matter, however, attempting to model the state-transition structure of a financial contract as a product automaton is awkward and ultimately counterproductive. For example, it does not generally matter to a contract if one party is secretly flirting with default, or the other is engaged in undetected activities that might (if caught and prosecuted) provoke mistrust. Modeling these sorts of party-specific states is typically a distraction. The agreement defines its own universe of relationship-specific states and events – which may include party-specific states where needed, for example as covenants, representations, and warranties.

The availability of a fully developed and well understood formalism such as the DFA for capturing the deep structure of financial agreements is likely to further energize the ongoing trend toward contract automation, discussed in Section III above. The fact that humans will retain a role, perhaps in the interpretation of input events or the state of the relationship, does not mean that the boundary between manual and automated tasks will not shift. Traditional tasks of lawyering will continue the march toward automation.[62] In this evolution, we should reserve for humans those tasks for which we retain an absolute or comparative advantage, such as those requiring nuanced judgment over legal or ethical dilemmas or strategic financial goals.

In contrast, although parsing the logic of a complex financial contract is traditionally among the hardest of the tasks of interpretation, the availability of a robust computational model will expose parts of that task to automation. For example, one might develop automated reasoners to discern whether a draft

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[61] See Hopcroft, J., R. Motwani, and J. Ullman. *Introduction to Automata Theory, Languages, and Computation*. 2nd ed. Boston: Pearson/Addison Wesley, 2001. 38-35.

[62] Engineering for human-in-the-loop systems is an established field, and there is an extensive literature on human-computer interaction (HCI). See, for example, M.G. Helander, T.K. Landauer, and P.V. Prabhu, eds., Sears, Andrew. *The Human-computer Interaction Handbook Fundamentals, Evolving Technologies, and Emerging Applications*. 2nd ed. New York: Lawrence Erlbaum Associates, 2007, or the articles in the ACM Transactions on Computer-Human Interactions (TOCHI), at: "Home." TOCHI. Accessed December 13, 2014. http://tochi.acm.org/. More generally, Hamid Ekbia and Bonnie Nardi refer to human-machine interdependence as "heteromation; see Ekbia, Hamid, and Bonnie Nardi. "Heteromation and Its (dis)contents: The Invisible Division of Labor between Humans and Machines." *First Monday* 19, no. 6 (2014). available at http://firstmonday.org/ojs/index.php/fm/article/view/5331).

agreement is complete in the sense of being able to accept certain key event sequences. Similarly, one might develop objective criteria for automatically scoring the computational or transactional complexity of a legal agreement, on either a relative (to other, similar agreements) or an absolute scale.

We emphasize that the DFA is a high-level system diagram setting out the logical structure of the contract. It is not intended to be a procedural flowchart for automating the relationship. [63] Indeed, the DFA would be a relatively cumbersome form for a computation engine, as each step must be set out in order, without the availability of memory or recursive operations to reduce the complexity of the representation. Actual implementation of more complex financial contracts will use these shortcuts. Nonetheless, the DFA is a good starting point for highlighting the conceptual link between contracting and computation.

An especially important extension of the state-transition model applies to financial agreements that interact through the events they consume. A finite-state transducer (FST) is an enhanced DFA that allows transitions to *emit* events. That is, contracts are not always "mere" consumers of events; contracts can generate events as well. Moreover, the output events for one agreement may be input events for another. A canonical example is a cross-default clause, which specifies that one contract should "listen" for transitions to default states as they occur in another agreement. Cross-default clauses can have systemic implications, because they typically trigger payments acceleration, thus creating a legal mechanism for the propagation of default across a network of contracts. A standard FST is a 6-tuple augments the basic DFA by equipping it with an output alphabet, often represented as Γ. The transition function, δ, is similarly augmented, so that a transition is associated with a character from both the input alphabet, Σ , and the output alphabet, Γ.[64] The FST model also may help represent how to deliver information about particular transitions to the parties themselves, even when there are not systemic consequences.

VII Implications and Conclusions

[63] At the implementation level, many contracts will also involve some calculation chores, such as determining precise payment amounts, sorting through holiday calendars and day-count conventions, etc. The "toy" agreement elides these considerations by explicitly stating precise dates and dollar amounts. In general, these sorts of calculations would and should typically be the implementation details of some delegated subsystem, and therefore need not detain us here.

[64] Transducers are widely used in control systems, including computer hardware design, and in computational linguistics. The control applications are closer to our case of computable contracts; see Mueller, Silvia M., and Wolfgang J. Paul. *Computer Architecture Complexity and Correctness*. Berlin: Springer, 2000. There are two general classes of FSTs, called Moore machines and Mealy machines, which differ essentially in whether outputs can (Mealy) or cannot (Moore) depend on the input event that triggered the transition. See Moore, Edward F (1956). "Gedanken-experiments on Sequential Machines". Shannon, Claude Elwood, and John McCarthy. *Automata Studies*. Vol. 34. Princeton: Princeton University Press, 1956. 129-153.; and Mealy, George H. "A Method for Synthesizing Sequential Circuits." *Bell System Technical Journal*, 1955, 1045-079. It remains a topic for future research which of these structures would best suit the analysis of computable contracts.

There are a number of potentially significant implications that flow from this exercise in computational contract specification. At the most basic level, the exercise of stating even a simplified contract as a DFA serves as a proof of concept, demonstrating that we can describe legal rule and consequence structures directly in computational terms. Of course, success here does not prove that this modeling technique would apply to all contracts, including those of much greater complexity. At this stage, however, we see no conceptual barrier to such a task, provided the contracts are reasonably bounded in their terms.

A practical complication that we consciously excluded from the scope of our "toy" contract was any sort internal governance mechanism to manage the relationship in the face of future uncertainty. Most financial contracts do not have internal governance provisions, and so can be expected to be susceptible to DFA representation. We note that the litigation exit specification in our example in effect moves the result determination out of the contract itself when faced with a court proceeding. This is a necessary step; such a proceeding constitutes just such a governance mechanism. As discussed below, tackling more complex contracts is a logical next step; our work here both suggests methods for doing that and provides hope that the effort involved will be time successfully invested.

The exercise of representing contracts as DFAs can help us better understand how contracts work. Seeing the graphical representation of the paths that flow from breaches of warranties and covenants, for instance, have helped us to conceptualize the role they play in financial contracts, effectively shifting the deal from a chain of original expectation and timing, to a new agreement that is generally time- and value-collapsed. We do not claim that such insights could not be gained by looking just at the natural language version, but they leap out from the graphical version.

There are implications for legal education. Law schools of the future would be well advised to teach "legal coding" as well as "legal writing." Acquiring such knowledge seems daunting to many of us now in the law, but it is probably less challenging – and more logically consistent – than having to learn the intricacies of Blue Book citation.

The DFA exercise also gives formal expression to our categorization of contractual functions into actions (states), events (alphabet), calculations and chains of logical application (transformations). This division helps to clarify where specificity and completeness is in fact a virtue, and in the process helps to resolve some of the apparent conflicts between ambiguity-rooted discretion and legal automation.

Taken a step further, using the DFA as a starting point to conceptualize and design a contract provides the drafter with a more controlled pallet for representing the logic of a transaction. Those of us who grew up before the spread of personal computing can recall the struggle to master the operating systems and user interfaces of the new machines.[65] Mastery of basic skills, such as launching programs and saving files, quickly progressed to higher-level challenges, such as configuring programs and organizing filesystem directories, etc. A similar transition awaits the contract drafters of the future, as such tools as promise, default and acceleration become deployable as explicit steps in a chain within a state machine. This vision of starting the drafting of contracts in formal computational representation

[65] The machines also struggled to adapt. The same generation of users will recall Microsoft Windows' infamous "blue screen of death," which signaled a total operating system crash, typically due to an untrapped exception.

suggests a different line of research from those looking to use learning algorithms or other parsing tools to abstract computable meaning from word based formulations.

Although embodying financial contracts in software has the potential to provide significant benefits, we also recognize that the increases in speed, accuracy and flexibility that this development will provide will have the potential to create problems as well. In an automotive analogy, a significant increase of power and acceleration in the motor probably needs to be balanced by increased efficiency in the brakes. The use of the use of one of the simplest computational formalisms (the DFA) in our "toy" example is intentional in this regard. There is a danger of the "Sorcerer's Apprentice" problem, that the unwise application of powerful computational tools could encourage inexperienced drafters to create contracts of unmanageable complexity.[66]

One of the more significant implications of this exercise is that the link between law and formal computation is not a one way street. If we can restate a natural language contract as a DFA as we have done here, then we may reasonably conclude that the natural language original was itself a statement of a DFA. The sometimes tortured prose of legalese, from this standpoint, can be viewed as enabling the use of natural language to create a word-based automaton. If this is so, perhaps we can turn the analytic tools of computation theory onto legal systems as they exist now, and not just on the formal representations that may contain our laws in the future. This extension of computational approaches into our natural language formulations suggests that computational analysis may extend beyond the law, into further aspects of human cognition, expression and behavior. Computational neuroscience, computational psychology, and computational economics are already fields of study, and computational law may have many companions as an applied science of thought and behavior.[67] Work in the existing field of natural language processing often borders on such research, but focuses more on algorithms for extracting meaning from natural language (a version of the feature extraction problem cited above) rather than on viewing natural language itself as a computational model.[68]

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[66] Computer systems designers worry actively about the problem of state space "explosion" (i.e., proliferation). See, for example, Baier, Christel, and Joost Katoen. *Principles of Model Checking*. Cambridge, Mass.: MIT Press, 2008, esp. ch. 2.

[67] See, e.g., Bower, James M. *20 Years of Computational Neuroscience*. New York, NY: Springer, 2013.; Sun, Ron. *The Cambridge Handbook of Computational Psychology*. Cambridge, UK: Cambridge University Press, 2008.; Tesfatsion, Leigh, and Kenneth Judd. *Handbook of Computational Economics*. Amsterdam: Elsevier, 2006.); and, generally, the series Schmedders, Karl. *Handbook of Computational Economics*. Amsterdam: North Holland, 2014., available at http://www.sciencedirect.com/science/handbooks/15740021, and "Computational Economics - Springer." Computational Economics - Springer. Accessed December 13, 2014. http://link.springer.com/journal/10614.

[68] See, e.g., Cambria, Erik, and Bebo Whitel. "Jumping NLP Curves: A Review of Natural Language Processing Research." *IEEE Computational Intelligence Magazine* 48 (2014).available at http://sentic.net/jumping-nlp-curves.pdf. But see, Liu, Hugo, and Henry Lieberman. "Metafor: Visualizing Stories as Code." 2005., part of the proceedings of IUI'05, January 10–13, 2005, San Diego, California, USA, available at http://web.media.mit.edu/~hugo/publications/papers/IUI2005-metafor.pdf

Representing a contract in computational terms also has connections with research in game theory and institutional design. It is possible to view the role of contracts and of many forms of legal, cultural and biological "institutions" more generally, as means of re-denominating the game structure of potentially cooperative interactions so as to move from predatory to mutually beneficial outcomes. [69] If this is so, and if a contract can be restated as a DFA, then we may also be able to represent the architecture of strategic games more generally in computational terms. [70] As we have suggested, contracts are a subset of institutions, in the sense of rule sets that re-denominate the strategic landscape of cooperation problems. The fact that at least some contracts can be restated as DFAs suggests that institutions more broadly – and perhaps the strategic structures of game theory generally – may be susceptible to similar treatment. A homology between game theory and linear programming has long been recognized.[71] This would help validate the approach, and would be a potential mine for design elements.[72]

The analysis of a simplified "toy" agreement initiates a larger project of automating financial instruments. An obvious next step will be to undertake such a description for existing agreements actually used in financial markets, such as the 2002 ISDA Master Agreement and the 1997 International Foreign Exchange Master Agreement. While the proof necessarily awaits the exercise, a preliminary review of these agreements suggests that the challenges of restating them as DFA are those of time and patience in the face of complexity, rather than of fundamental differences of kind. Such a step would be a precursor to creating a software version of these instruments; it could also suggest re-drafting opportunities. We can imagine an updated "2016 ISDA Master Agreement" that is completely computable. In addition to restating existing natural language contracts as DFA, another plausible next

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[69] See, e.g., Zak, Paul J. *Moral Markets the Critical Role of Values in the Economy*. Princeton: Princeton University Press, 2008.

[70] Nisan, Noam, Vijay V. Vazirani, Tim Roughgarden, and Eva Tardos. *Algorithmic Game Theory*. Cambridge: Cambridge University Press, 2007. http://www.cambridge.org/journals/nisan/downloads/Nisan_Non-printable.pdf; Dütting, Paul, and Andreas Geiger. "Seminar Report: Algorithmic Mechanism Design." May 9, 2007. Accessed December 13, 2014. http://www.cs.uu.nl/docs/vakken/msagi/mech_design.pdf. and Nisan, Noam. "Algorithmic Mechanism Design: Through the Lens of Multi-unit Auctions." January 21, 2014. Accessed December 13, 2014. http://www.cs.huji.ac.il/~noam/amd-mua.pdf.

[71] Brickman, Louis. *Mathematical Introduction to Linear Programming and Game Theory*. New York: Springer-Verlag, 1989.; Thie, Paul R., and G. E. Keough. *An Introduction to Linear Programming and Game Theory*. 3rd ed. Hoboken, N.J.: Wiley, 2008.. Harold W. Kuhn, in the introduction to the 2004 reissue of the Theory of Games notes, "By the end of the summer [of 1948], we had established that, mathematically, linear programming and the theory of zero-sum two-person games are equivalent." Neumann, John, and Oskar Morgenstern. *Theory of Games and Economic Behavior*. 60th Anniversary ed. Princeton: Princeton University Press, 2007. http://press.princeton.edu/chapters/i7802.pdf.

[72] Fernando Bevilacqua notes that, "aspects of agent-based behavior have been modeled in state machine terms." See "Finite-State Machines: Theory and Implementation - Tuts Game Development Tutorial." Game Development Tuts. October 24, 2013. Accessed December 13, 2014. http://gamedevelopment.tutsplus.com/tutorials/finite-state-machines-theory-and-implementation--gamedev-11867.

step would be a project of new drafting, where the goal is to embody transactional structures straight into the formalism of computation and software, without relying on a natural language precursor. Computational drafting is likely to be a necessary skill for some lawyers of the future.

The DFA representation of financial contracts should also assist in additional projects of the OFR and others seeking to make our financial firms and markets more robust in the face of risk and disruption. For instance, the OFR is working to create a library of contingency clauses commonly used in financial instruments. The frequent function of such clauses, particularly those linked to events of termination or default, is to change the pathways of result from one anticipated chain of calculation and performance to another. DFA representations, particularly the graphical approach, can help us clarify these pathways and increase our understanding.

Another possible application of the state-transition approach is to restate other legal formulations such as statutes and regulations in DFA terms. One possible target is Regulation D under the Securities Act of 1933. Reg. D sets out a safe harbor from the registration requirements of the 1933 Act for private placement transactions. It is relatively self-contained, and it appears that the cross references with the larger world of SEC regulation and beyond can be handled through event definition shortcuts in the feature extraction exercise. A broader target might include the Federal Rules of Civil Procedure; it is interesting to speculate on whether the procedural aspect generally of a formal litigation is itself describable as a state machine. In this sense, a summary judgment proceeding or even a full trial might also be seen as a computation, with the states and transitions perhaps defined by the elements of the cause of action and the rules of evidence and procedure, and with the alphabet provided by the evidence presented.

As this discussion demonstrates, the application of computational approaches to legal formulation, whether in contracts or more broadly, is not just a matter of metaphor. The tools of computation theory have direct application in the specification of a system of ordered and defined rights and obligations. Putting computational tools to work in the context of law and justice has the potential to revolutionize aspects of the legal system, with significant consequences for markets, government, and society. Law is getting “Turing’d”; it is time to recognize the fact and to work to make the process as beneficial as possible.

TABLES

Table 1: Ingredients of a typical non-equity-style contract

Counterparties	A specification of the parties to the agreement and the date of its formation and effectiveness; Specified means of communication between the parties;
Basic obligations	One or more obligations for payment, transfer or other financial performance, together with the calculations that specify the amount or extent of this obligation; A fixed date, or means for fixing the date, for the execution of the obligations for payment, transfer or other financial performance; A calculation of interest, exchange, or some other rate of return; Covenants, which provide promises of things which a party is obligated to do or obligated to refrain from doing, generally not including the underlying financial obligations described above; Warranties, which provide a promise about a state of facts; these have the effect of creating factual parameters for the transaction and of allocating the risks for a failure of the facts to conform to those parameters; The provision of security, either through a guaranty by another entity or through the dedication of some asset as a source of value if the transaction goes into default;
Default provisions	A procedure for declaring a default, which can be based on a failure to meet the obligations of performance, including the covenants, a failure of a warranty to be accurate either when made or on a continuing basis, or other specified events, such as bankruptcy or the default under other obligations (cross-default); Additional rights granted the "innocent" party in the case of a default, including the acceleration of obligations and the authorization of protective measures (these rights in effect re-denominate the contract); The imposition of penalties, including increased and continuing rates of interest, in the case of a default; Avenues for proceeding against the security of collateral or guaranty; Procedures for going forward if certain specified or unanticipated events disrupt the transaction (force majeure);
Enforcement	A procedure for resolving disputes between the parties (courts, arbitration, etc.); A procedure for enforcing the obligations between the parties; A procedure for making the non-defaulting party whole (indemnity) for the costs of invoking those procedures; A specification of what legal regime will govern the transaction (choice of law); and A procedure for waiver, amendment, renegotiation and agreed termination of the agreement.

Table 2: A "Toy" Loan Agreement

Agreement

This loan agreement dated June 1, 2014, by and between Lender Bank Co. ("Lender") and Borrower Corp. (Borrower), will set out the terms under which Lender will extend credit in the principal amount of $1,000 to Borrower with an un-compounded interest rate of 5% per annum, included in the specified payment structure.

1. The Loan:
At the request of Borrower, to be given on June 1, 2014, Lender will advance $1000 to Borrower no later than June 2, 2014. If Borrower does not make such a request, this agreement will terminate.

2. Repayment:
Subject to the other terms of this agreement, Borrower will repay the loan in the following payments:

(a) Payment 1, due June 1, 2015, in the amount of $550, representing a payment of $500 as half of the principal and interest in the amount of $50.

(b) Payment 2, due June 1, 2016, in the amount of $525, representing a payment of $500 as the remaining half of the principal and interest in the amount of $25.

3. Representations and Warranties:
The Borrower represents and warrants, at the execution of this agreement, at the request for the advance of funds and at all times any repayment amount shall be outstanding, the Borrower's assets shall exceed its liabilities as determined under an application of the FASB rules of accounting.

4. Covenants:
The Borrower covenants that at the execution of this agreement, at the request for the advance of funds and at all times any repayment amount shall be outstanding it will make timely payment of all state and federal taxes as and when due.

5. Events of Default:
The Borrower will be in default under this agreement upon the occurrence of any of the following events or conditions, provided they shall remain uncured within a period of two days after notice is given to Borrower by Lender of their occurrence (such an uncured event an "Event of Default"):

(a) Borrower shall fail to make timely payment of any amount due to Lender hereunder;

(b) Any of the representation or warranties of Borrower under this agreement shall prove untrue;

(c) Borrower shall fail to perform any of its covenants under this agreement;

(d) Borrower shall file for bankruptcy or insolvency under any applicable federal or state law.

A default will be cured by the Borrower (i) remedying the potential event of default and (ii) giving effective notice of such remedy to the Lender. In the event of multiple events of default, the first

to occur shall take precedence for the purposes of specifying outcomes under this agreement.

6. Acceleration on Default

Upon the occurrence of an Event of Default all outstanding payments under this agreement will become immediately due and payable, including both principal and interest amounts, without further notice, presentment, or demand to the Borrower.

7. Choice of Law:

This agreement will be subject to the laws of the State of New York applicable to contracts entered into and performed wholly within that state.

8. Amendments and Waivers:

Any purported amendment to, or waiver of rights under, this agreement will only be effective if set forth in writing and executed by both parties.

9. Courts and Litigation:

Any legal action brought to enforce, interpret or otherwise deal with this agreement must be brought in the state courts of the State of New York located in New York County, and each of the parties agrees to the jurisdiction of such courts over both the parties themselves and over the subject matter of such a proceeding, and waives any claim that such a court may be an inconvenient forum.

10. Time of the Essence; No Pre-Payment

Timely performance is required for any action to be taken under this agreement, and, except as may otherwise be specifically provided herein, failure to take such action on the day specified will constitute a binding failure to take such action. Payments shall only be made on or after the dates specified in Section 2 or on or after such other date as may be required under Section 6; pre-payments made on earlier dates shall not be accepted.

11. Notices

Notices provided for in this agreement will be given by an email to the email addresses set out below and will be effective upon receipt.

[Lender email here]

[Borrower email here]

Accepted and agreed:

LENDER BANK CO.	BORROWER CORP.
By: _____	By: _____
Title: _____	Title: _____

[NOTE: Statute of Limitations on debt obligations in NY is 6 years]

Draft of July 23, 2014

Table 3: Contract States (Q)

State	Label	Natural Language Consequences and Correlates (Λ)	Sec
start ☺	Start	Contract is fully specified; key information (payment dates, notice addresses and procedures, choice of law and dispute process) delivered	7, 9, 11
q0 ☺	Active contract	Contract is fully signed/executed	
q1 ☺	Principal Requested	Borrower's has requested and awaits $1,000	1
P1 ☺	Pmt 1 accruing		
P1d ☺	Pmt. 1 due		
P2 ☺	Pmt 2 accruing		
P2d ☺	Pmt. 2 due		
Dl	Default (lender)	Lender has failed to deliver principal	5
Acc1	Payments 1 and 2 Accelerating	Accelerated payment due is $1075	6
Acc2	Payments 2 Accelerating	Accelerated payment due is $525	6
Db0_1	Default (borr.) payment missed	Borrower has failed to make first payment in a timely fashion, and should be notified	5
Dbcv_1	Default (borr.) Covenant	Borrower violates covenant(s), and should be notified	5
Dbrw_1	Default (borr.) Reps/Warr.	Borrower breaches reps. and/or warranties; should be notified	5
Dbbkr_1	Default (borr.) Bankruptcy	Borrower files for bankruptcy or insolvency; should be notified	5
Nb0_1 Δ	Borr. notified of payment default	Borrower has 2 days to pay, or all payments accelerate	5
Nbnpd_1 Δ	Borr. notified of general default	Borrower has 2 days to pay, or all payments accelerate	5
Db0_2	Default (borr.) payment missed	Borrower has failed to make first payment in a timely fashion, and should be notified	5
Dbcv_2	Default (borr.) Convenant	Borrower violates covenant(s), and should be notified	5
Dbrw_2	Default (borr.) Reps/Warr.	Borrower breaches reps. and/or warranties; should be notified	5
Dbbkr_2	Default (borr.) Bankruptcy	Borrower files for bankruptcy or insolvency; should be notified	5
Nb0_2 Δ	Borr. notified of payment default	Borrower has 2 days to pay, or all payments accelerate	5
Nbnpd_2 Δ	Borr. notified of general default	Borrower has 2 days to pay, or all payments accelerate	5
xT †	TERM	Contract is fulfilled in accordance with its terms	
xL †	LIT	A legal action is brought to enforce, interpret or otherwise deal with the agreement in the state courts of the State of New York located in New York County and that the results of this action will replace the computation of the contract	9
xC †	CANC	Contract is canceled due to the passing of time beyond the statute of limitations or canceled because of modification or termination by mutual agreement of the parties	8
Crisis1	Crisis1 – accel. payments not made	Payments accelerated, but borrower has not responded	6
Crisis2	Crisis2 – accel. payments not made	Payments accelerated, but borrower has not responded	6

☺ *States on the "happy" path of the contract lifecycle*
Δ *Default states*
† *Terminal states*

Table 4: Event Alphabet (Σ)

ID	Label	Natural Language Event Specification	Section
A	Contract signed	Contract is signed to bind all parties	
B	1 Day passes since last event	June 1, 2014 passes	1
C	Money requested	Borrower gives request for loan of $1,000	1
D	Lawsuit	A legal action is brought to enforce, interpret or otherwise deal with the agreement in the state courts of the State of New York located in New York County.	
E	Statute of limitations	June 1, 2020 passes – the Statute of Limitations on debt obligations in New York is six years	
F	Principal advanced	Lender advances $1,000 no later than June 2, 2014	1
G	June 1, 2015 passes	Payment 1 due on June 1, 2015	2(a)
H	Reps/warr.	The Borrower's assets exceed its liabilities as determined under an application of the FASB rules of accounting	3, 5(b)
I	Covenant	The Borrower fails to make a timely payment of an amount of state or federal tax	4, 5(c)
J	Bankruptcy	The Borrower files for bankruptcy or insolvency under any applicable federal or state law	
K	Notice given	Notice given to Borrower of a failure to make timely payment of an amount due to Lender under this agreement	5
L	Notice given of general default	Notice given to Borrower of an event of default other than a failure to make timely payment of an amount due	5
M	Payment default cured	A payment-related event of default is cured	5
N	General default cured	A non-payment related event of default is cured	5
O	2 Days pass since last event	Two days elapse since last event/notice	5
P	June 1, 2016 passes	Payment 2 is due on June 1, 2016	2(b)
Q	Payment made $550		
R	Payment made $525		
S	Payment made $1075		
T	Cancel or modify	Contract in this form is canceled because of modification or termination by mutual agreement of the parties	8

Table 5: Transition Function (δ)

Initial State	Event	Resulting State
start ☺	A	q0
q0	B	xT
q0 ☺	C	q1
q1	B	Dl
Dl	D	xL
Dl	E	xC
q1 ☺	F	P1
P1 ☺	G	P1d
P1d	B	Db0_1
P1	H	Dbrw_1
P1	I	Dbcv_1
P1	J	Dbbkr_1
Db0_1	K	Nb0_1
Dbcv_1	L	Nbnpd_1
Dbbkr_1	L	Nbnpd_1
Dbrw_1	L	Nbnpd_1
Nb0_1	Q	P2
Nbnpd_1	N	P1
Nb0_1	O	Acc1
Nbnpd_1	O	Acc1
Acc1	B	Crisis1
Acc1	S	xT
Crisis1	E	xC
Crisis1	D	xL
Crisis1	S	xT
P1d ☺	Q	P2

(Continued)

Initial State	Event	Resulting State
P2 ☺	P	P2d
P2d	B	Db0_2
P2	H	Dbrw_2
P2	I	Dbcv_2
P2	J	Dbbkr_2
P2d ☺	R	xT
Db0_2	K	Nb0_2
Dbcv_2	L	Nbnpd_2
Dbbkr_2	L	Nbnpd_2
Dbrw_2	L	Nbnpd_2
Nb0_2	R	xT
Nbnpd_2	N	P2
Nb0_2	O	Acc2
Nbnpd_2	O	Acc2
Acc2	B	Crisis2
Acc2	R	xT
Crisis2	E	xC
Crisis2	D	xL
Crisis2	R	xT

☺ *Transitions along the "happy" path of the contract lifecycle*

As noted above, only the transitions that result in a change of states are noted here; all un-noted transitions result in the state being unchanged.

Table 6: Full Transition Matrix

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
start	q0	---	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	xC
q0	---	xT	q1	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	xC
q1	---	Dl	---	xL	---	P1	---	---	---	---	---	---	---	---	---	---	---	---	---	xC
P1	---	---	---	xL	---	---	P1d	Dbrw_1	Dbcv_1	Dbbkr_1	---	---	---	---	---	---	---	---	---	xC
P1d	---	Db0_1	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	P2	---	---	xC
P2	---	---	---	xL	---	---	---	Dbrw_2	Dbcv_2	Dbbkr_2	---	---	---	---	---	P2d	---	---	---	xC
P2d	---	Db0_2	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	xT	---	xC
Dl	---	---	---	xL	xC	---	---	---	---	---	---	---	---	---	---	---	---	---	---	xC
Acc1	---	Crisis1	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	---	xT	xC
Acc2	---	Crisis2	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	xT	---	xC
Db0_1	---	---	---	xL	---	---	---	---	---	---	Nb0_1	---	---	---	---	---	---	---	---	xC
Dbcv_1	---	---	---	xL	---	---	---	---	---	---	---	Nbnpd_1	---	---	---	---	---	---	---	xC
Dbrw_1	---	---	---	xL	---	---	---	---	---	---	---	Nbnpd_1	---	---	---	---	---	---	---	xC
Dbbkr_1	---	---	---	xL	---	---	---	---	---	---	---	Nbnpd_1	---	---	---	---	---	---	---	xC
Nb0_1	---	---	---	xL	---	---	---	---	---	---	---	---	---	---	Acc1	---	P2	---	---	xC
Nbnpd_1	---	---	---	xL	---	---	---	---	---	---	---	---	---	P1	Acc1	---	---	---	---	xC
Db0_2	---	---	---	xL	---	---	---	---	---	---	Nb0_2	---	---	---	---	---	---	---	---	xC
Dbcv_2	---	---	---	xL	---	---	---	---	---	---	---	Nbnpd_2	---	---	---	---	---	---	---	xC
Dbrw_2	---	---	---	xL	---	---	---	---	---	---	---	Nbnpd_2	---	---	---	---	---	---	---	xC
Dbbkr_2	---	---	---	xL	---	---	---	---	---	---	---	Nbnpd_2	---	---	---	---	---	---	---	xC
Nb0_2	---	---	---	xL	---	---	---	---	---	---	---	---	---	---	Acc2	---	---	xT	---	xC
Nbnpd_2	---	---	---	xL	---	---	---	---	---	---	---	---	---	P2	Acc2	---	---	---	---	xC
xT	---	---	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	xC
xL	---	---	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	xC
xC	---	---	---	xL	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	xC
Crisis1	---	---	---	xL	xC	---	---	---	---	---	---	---	---	---	---	---	---	---	xT	xC
Crisis2	---	---	---	xL	xC	---	---	---	---	---	---	---	---	---	---	---	---	xT	---	xC

FIGURES

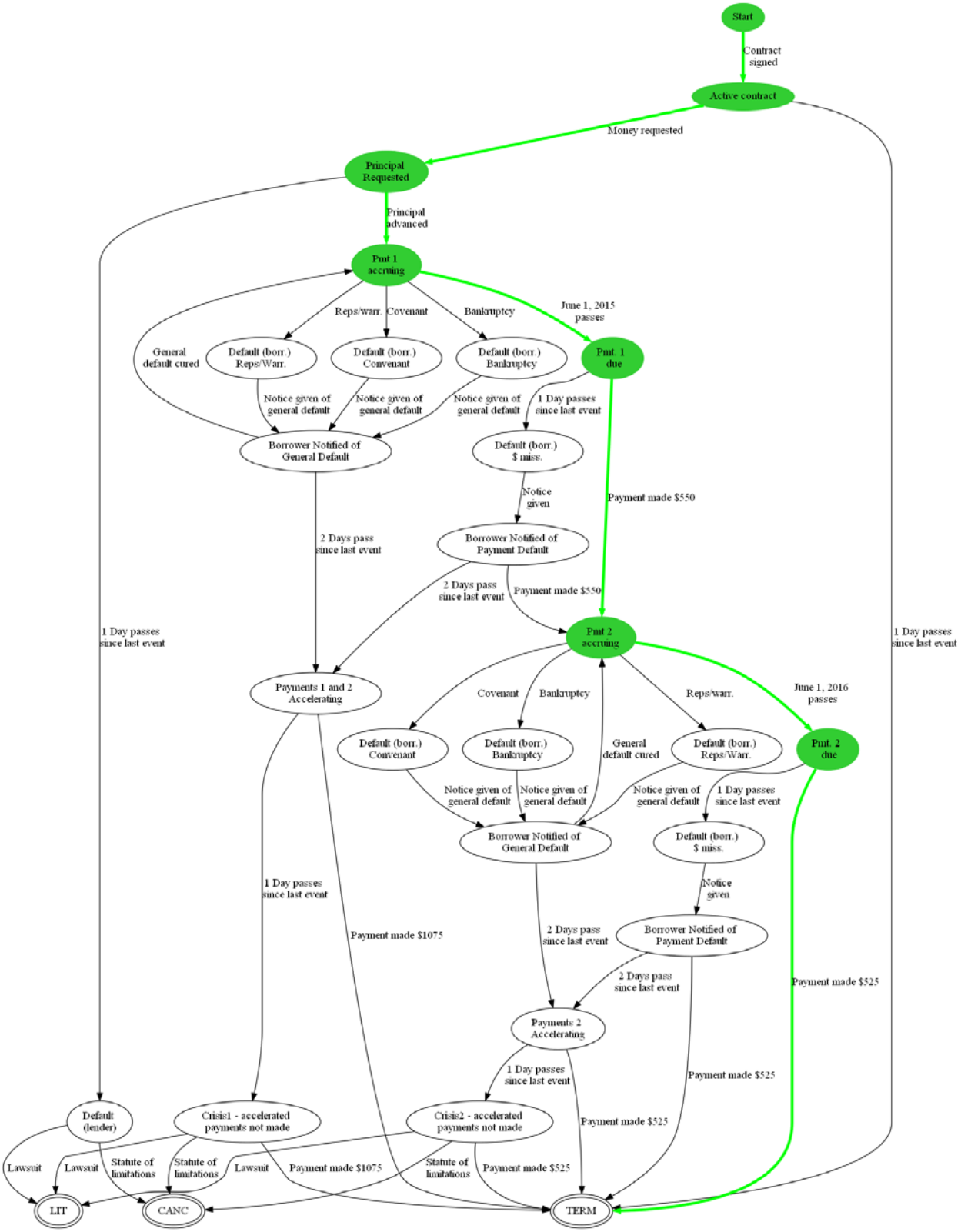

Figure 1: Graphical representation of the DFA for the "toy" contract