Grey Swans

Plausible Stress Testing

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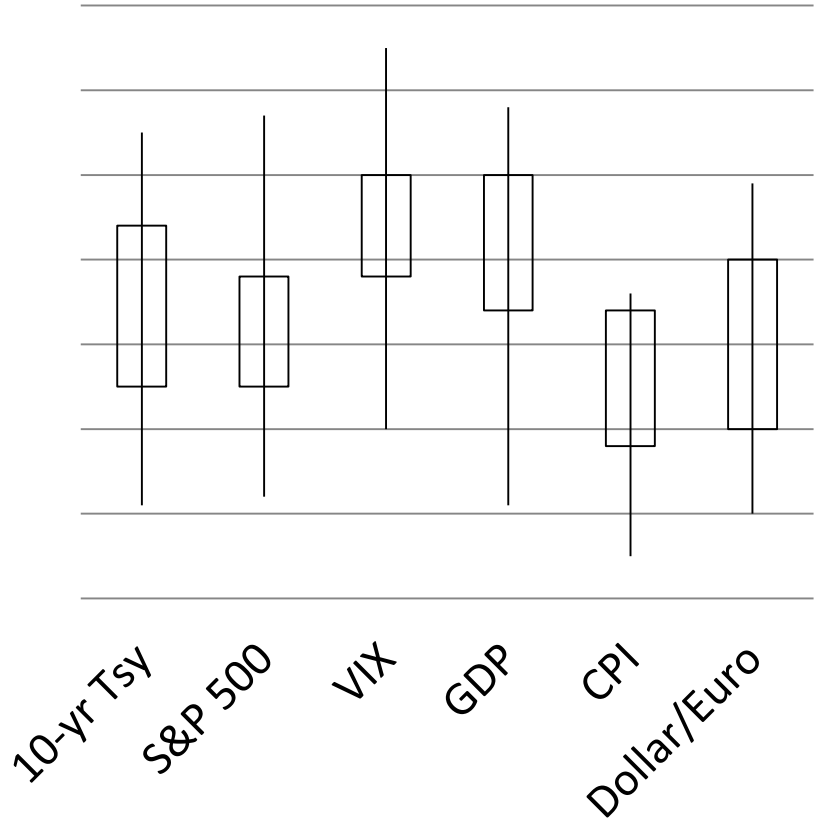
Abstract

For the purposes of risk management and stress testing we characterize a spectrum of plausible extreme events, that we dub ‘Grey Swans’, by introducing a probabilistic method involving the concentration of measure phenomenon.

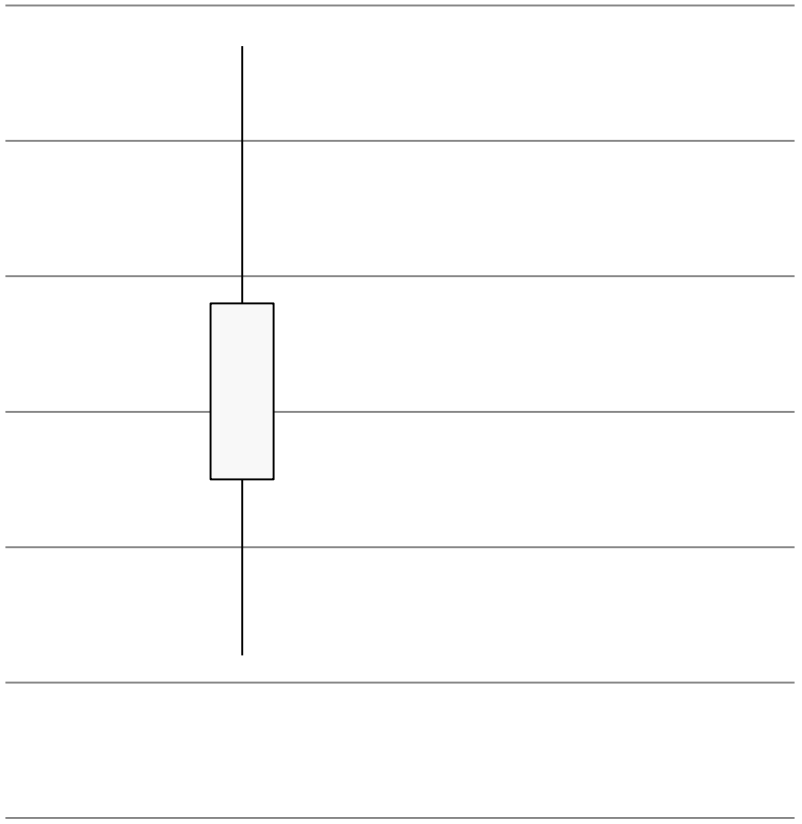
As a result, stress scenarios can be designed according to their (information theoretic) **plausibility of pattern**, in addition to their severity and probability of outcome.

Establish a baseline stress scenario by reverse stress test

Marginal Distributions of X



Portfolio value V(x)



How plausible is a double, triple,… whammy?

- Let A be a large (observed or simulated) sample of economic scenarios from random vector

  X = (10-yr Tsy, S&P 500, VIX, GDP, CPI, €/$)

- Baseline stress scenario y ϵ A chosen so that portfolio value V(y) = 30th percentile, for example

- We will create ‘Grey Swans’, plausible n-fold whammy scenarios x outside of A, yet similar in pattern to y in A, such that V(x) < V(y)

Admissible scenario patterns

- In designing a stress test it is tempting to immediately conjecture a ‘perfect storm’. Notice however that only certain combinations of the macrovariables can arise. Extremeness/rareness of stress scenarios can be quantified by Mahalanobis distance, but what about their patterns of occurrence?
- Heuristically, we can enlarge A by adding strings that differ from those in A in at most one entry creating $A_1 \supset A$. Similarly we can next add strings that differ from those in A in at most two entries creating $A_2 \supset A_1 \supset A$ and so on.
- Technically, Talagrand’s convex distance allows us to define these enlargements A_t, and Talagrand’s concentration of measure inequality to quantify their plausibility. (Talagrand’s distance sandwiches Hamming distance.)
- So stress scenarios can be designed with economic patterns of varying plausibility.

Example

Let scenario x outside of A be a modification of y in A,

where we have replaced 3 components with ‘worse’ values, ‘Grey Swans’, (depending on which way the marginal is correlated to V), so that V(x) < V(y). The choice of which macrovariables and their values will depend on our utility for their severity and probability. This $x \in A_3$, similar in pattern to $y \in A$, is a triple whammy stress scenario.

If in the product space of marginals Pr[A] > 50%,

then Talagrand’s inequality implies $Pr[A_3] > 78.9\%$. That is to say, this enlargement contains a majority of the economic patterns similar to those in A. So scenarios outside of A_3 can be considered implausible, not being consistent with most patterns. In this sense, it is plausible to have up to 3 extreme events (Grey Swans) occurring at once but not more.

Summary

- In addition to the usual considerations of severity and probability in designing stress tests, we may also take into account the **plausibility of their economic pattern**. We can thus focus on mitigating the most plausible and most probable extreme scenarios.

References:

Grey Swans: Fifty Shades of Grey
Plausible Stress Testing
Private Communication, Gary Nan Tie, September 4, 2014

Talagrand, Michel (1995). 'Concentration of measure and isoperimetric inequalities in product spaces.' Publications Mathematiques de l'IHES, Dec. 1995, Vol 81, Issue 1, pp. 73-205.