Statistical Measures of Instability and Systemic Risk

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Financial turbulence as a measure of market instability

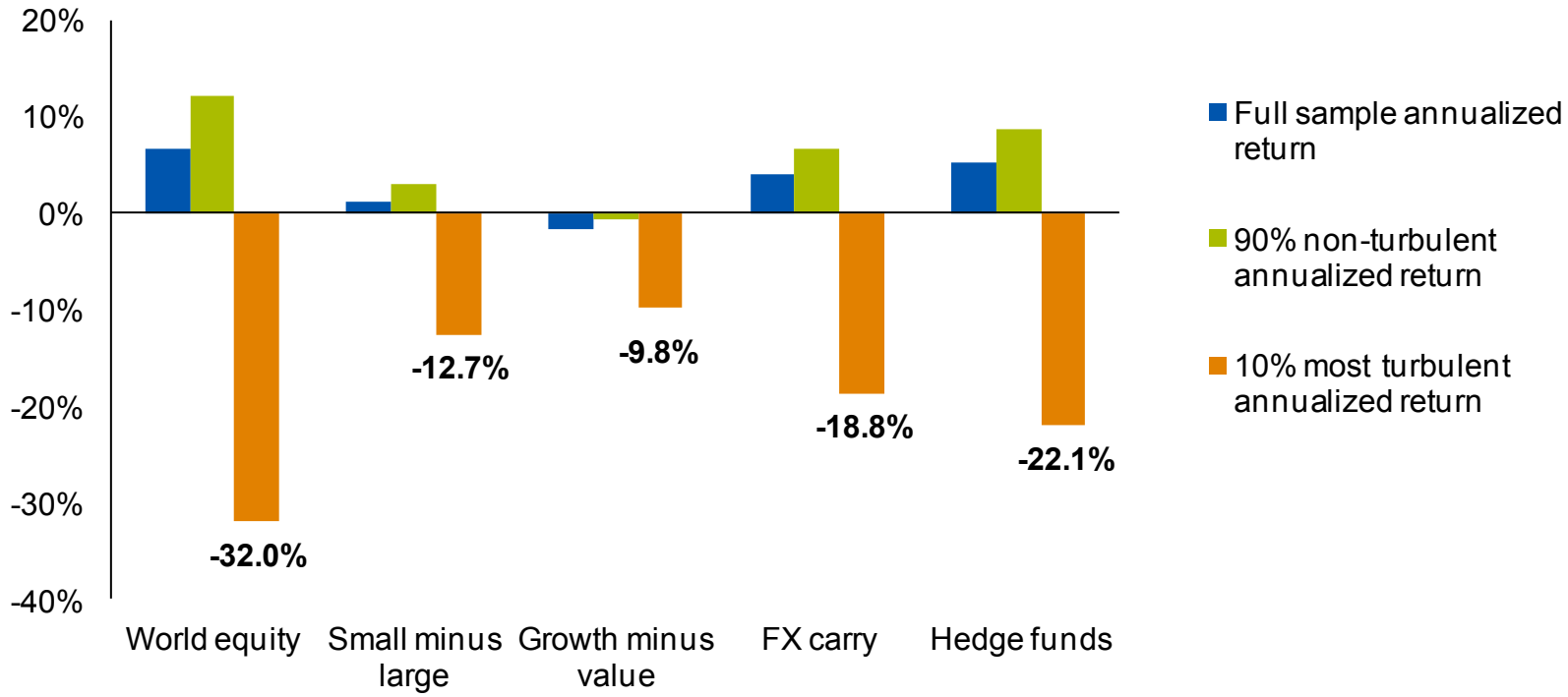
$$d_t = \frac{(y_t - \mu)\Sigma^{-1}(y_t - \mu)'}{N}$$

d_t = vector distance from multivariate average
y_t = vector of cross-sectional asset returns
μ = mean vector of return series *y*
Σ = covariance matrix of return series *y*
N = number of assets

- Turbulence measures the statistical unusualness of a cross-section of returns given their historical pattern of behavior. It captures:
  - Extreme price moves
  - The convergence of uncorrelated assets
  - The decoupling of correlated assets.
- Turbulence can be decomposed into two components:
  - Volatility surprise
  - Correlation surprise

For more details, please refer to Kritzman, Mark and Yuanzhen Li. 2010. "Skulls, Financial Turbulence, and Risk Management." *Financial Analysts Journal*, vol. 66, no. 5, (September/October): 30-41.

Financial turbulence and market performance



Turbulent periods are identified using USD-denominated daily values of the Turbulence Index constructed for Global Asset Allocation (World Equity), US Sectors (Size Premium and Value Premium), and Developed Currencies (Carry) over the time period 4 January 1993 through 1 April 2011 Monthly Turbulence Index values for Global Asset Allocation over the period January 1993 through February 2011 are used for Hedge Funds. Raw turbulence values are multivariate distances using a full-sample covariance matrix. The market returns are daily returns of MSCI World (World Equity), Russell 2000 minus S&P 500 (Size Premium), Russell 1000 Value minus Russell 1000 Growth (Value Premium), and a naïve carry strategy over the same time period. Monthly hedge fund returns are from HFRI fund of funds composite.

Source: State Street Associates

Why it is difficult to observe systemic risk directly

- Securitization obscures connections among stakeholders.
- Private transacting leads to opacity.
- Complexity reduces clarity.
- “Flexible accounting” also hides financial linkages.
- Even if we could identify the relevant linkages, they do not remain constant.

The absorption ratio as a measure of systemic risk

$$AR = \frac{\sum_{i=1}^{n} \sigma_{E_i}^2}{\sum_{j=1}^{N} \sigma_{A_j}^2}$$

AR: absorption ratio

N: number of assets

n: number of eigenvectors used to calculate AR

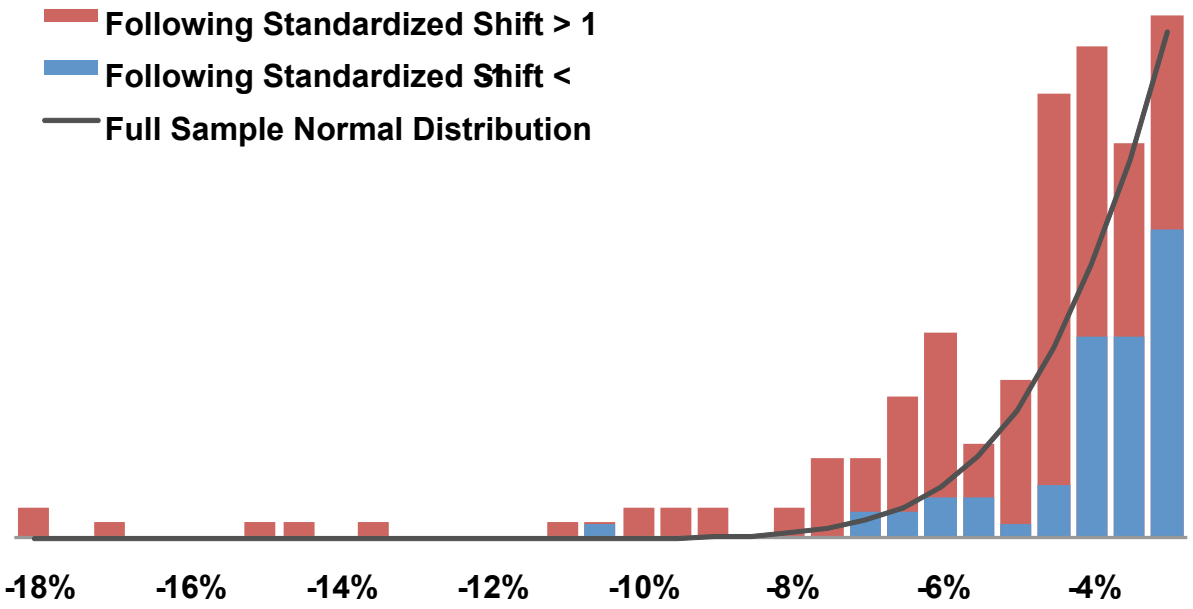
$\sigma_{E_i}^2$: variance of the *i*-th eigenvector

$\sigma_{A_j}^2$: variance of the *j*-th asset

- The absorption ratio equals the fraction of the total variance of a set of assets explained or “absorbed” by a finite number of eigenvectors.
- A high absorption ratio implies that markets are compact or tightly coupled.
- Compact markets are relatively fragile in that shocks propagate more quickly and broadly than when markets are loosely linked.

For more details, please refer to Kritzman, Mark, Yuanzhen Li, Sebastien Page, and Roberto Rigobon. 2011. “Principal Components as a Measure of Systemic Risk.” *The Journal of Portfolio Management*, vol. 37, no. 4 (Summer): 112-126.

Conditional distribution of one-week U.S. equity returns



The curved line shows the 10th percentile left tail, assuming normality and given the empirical mean and standard deviation of the full sample of the U.S. equity returns from January 1998 through June 2011.

Centrality as a measure of systemic importance

We can extend the absorption ratio to determine an asset's centrality.

Centrality captures three features:

- an asset's vulnerability to failure,
- how broadly and deeply an asset is connected to other assets in the system, and
- the riskiness of the other assets to which it is connected.

For more information, please refer to: Kinlaw, W., M. Kritzman and D. Turkington. 2012. "Toward Determining Systemic Importance." *The Journal of Portfolio Management*, vol. 38, no. 4 (summer):100-111.

Centrality

$$Asset\ Centrality_i = \frac{\sum_{j=1}^{n}\left(AR^j \cdot \frac{|EV_i^j|}{\sum_{k=1}^{N}|EV_k^j|}\right)}{\sum_{j=1}^{n} AR^j}$$

Normalized component "weights" in each eigenvector

$Asset\ Centrality_i$ = the centrality score for asset i

AR^j = the absorption ratio of the j-th eigenvector (percentage of variation explained)

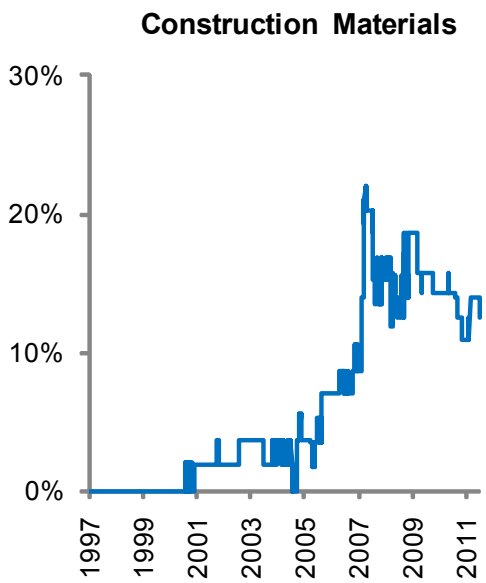
EV_i^j = the absolute value of the exposure of the i-th asset within the j-th eigenvector

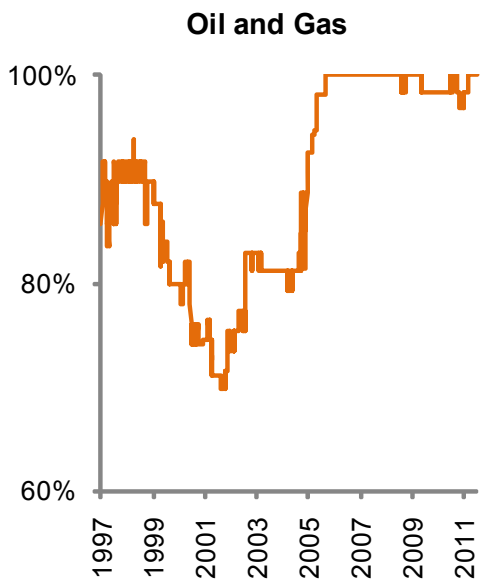
n = the number of top eigenvectors to include in the calculation

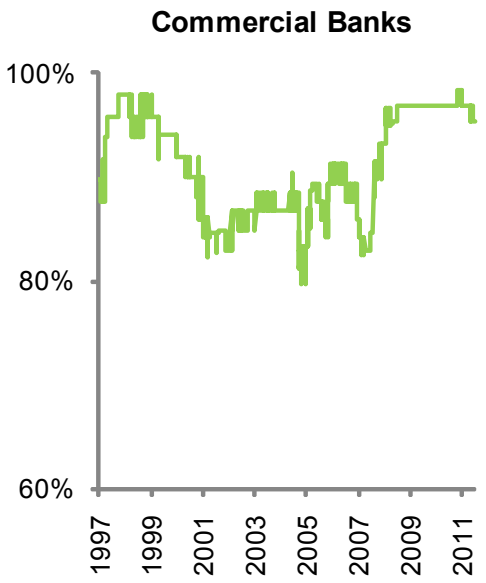
N = the total number of assets

The importance of each factor (eigenvector) is determined by its associated eigenvalue. An asset's centrality score is therefore the eigenvalue-weighted sum of the asset's weights in the eigenportfolios that represent the top factors.

Centrality percentile ranks for selected U.S. industries



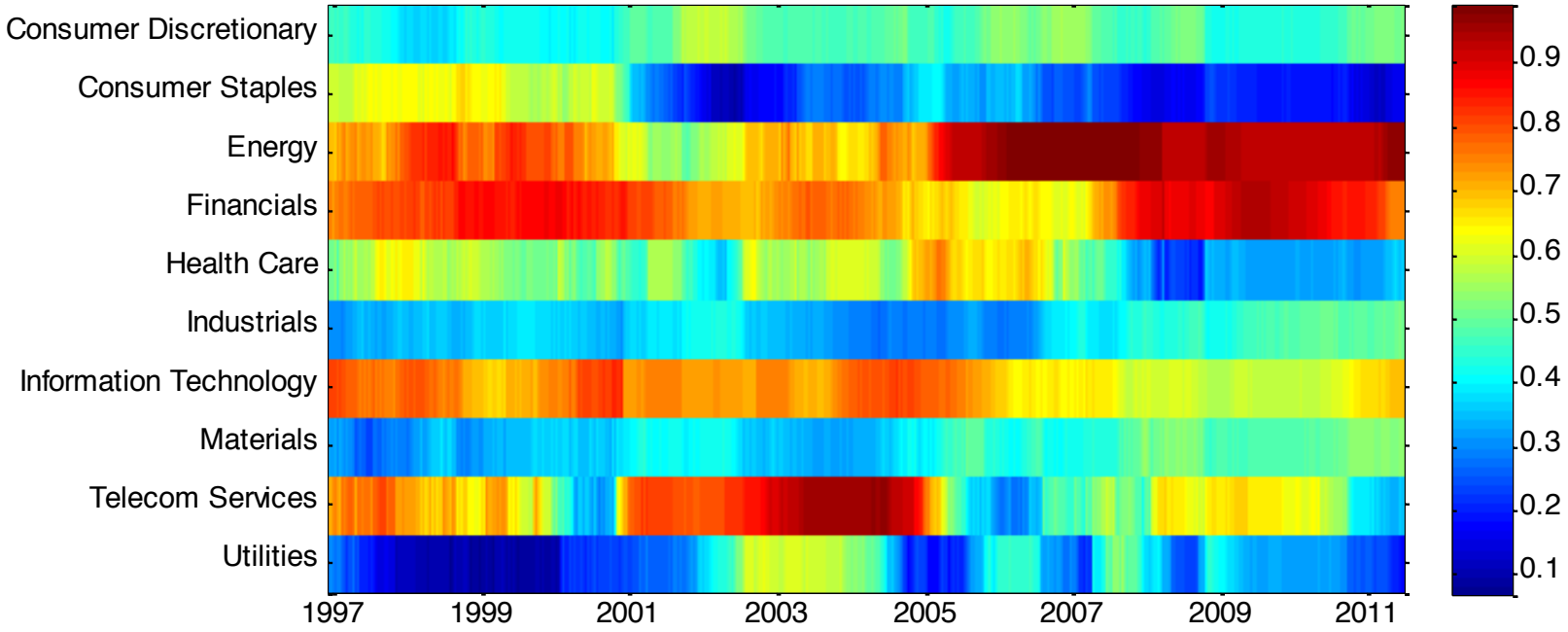




Dates: 1/1/1997-6/30/2011

Source: State Street Associates

U.S. sector centrality percentile ranks



Dates: 1/1/1997-6/30/2011

Source: State Street Associates

Top 25 systemically important financial institutions (as of Dec 2009)

Financial institutions	Rank
Bank of America	1
JP Morgan Chase	2
Wells Fargo	3
Citigroup	4
Barclays	5
Royal Bank of Scotland	6
HSBC	7
Lloyds Banking Group	8
BNP Paribas	9
Goldman Sachs	10
Morgan Stanley	11
Santander	12
US Bancorp	13

Financial institutions (cont'd)	Rank
UBS	14
ING	15
AXA	16
Unicredit	17
Mitsubishi UFJ FG	18
Credit Suisse	19
Met Life	20
Prudential Financial	21
Societe Generale	22
AIG	23
Deutsche Bank	24
Credit Agricole	25

While centrality identifies many of the same systemically important financial institutions as the Financial Stability Board (FSB), it requires data that are much easier to observe and collect than the FSB's methodology and generates results in much shorter time—one week versus two years.

Also appears on the FSB's list of 29 systemically important institutions

Company names provided above are for illustrative purposes only and do not represent an analysis of the merits of investing in such companies.
The universe is based on the MSCI Developed World Financials Sector index constituents as of November 2011. We removed companies from the Real Estate industry group as well as 22 companies that did not have a long enough stock price data history to include in our study.

Source: State Street Associates, Financial Stability Board